From the Tomash Library on the History of Computing

Napier, John (1550–1617) *Rabdologiae*, 1617, Edinburgh

Rabdologiae seu numerationis per virgulas libri duo: Cum appendice de expeditissimo multiplicationis promptuario. Quibus accessit & arithmeticae localis liber unus.

Year: 1617
Place: Edinburgh
Publisher: Andrew Hart
Edition: 1st
Language: Latin
Figures: 4 folding plates
Binding: original vellum; small spine tears
Pagination: pp. [12], 154, [2]
Collation: $¶^{6}A–F^{12}G^{6}$
Size: 145x75 mm
Reference:

Macdonald, William Rae, translator; *The construction of the wonderful Canon of Logarithms by John Napier translated from the Latin into English with notes and a Catalogue of the various editions of Napier's works*, Edinburgh, William Blackwood and Sons, 1889, pp. 131

Wing, Donald; *Short-Title Catalogue of Books printed in England, Scotland, Ireland, Wales and British America, and of English Books printed in other Low Countries 1641-1700*, New York, Columbia University Press, 1951, 18357

Notes on John Napier and the book

John Napier was born into a leading, prominent family of Scottish lairds (wealthy landowners). The family surname is seen in early documents as Napeir, Nepair, Nepeir, Neper, Napare, Naper, Naipper and the present-day Napier. Little is known about John Napier's childhood and youth. He enrolled at St. Andrews University at the age of thirteen, but there is no record that he ever graduated. Napier later wrote that his fervent interest in theology was kindled at St. Andrews. It is probable that he left St. Andrews to study in Europe, and it must have been there that he acquired his knowledge of higher mathematics and his taste for classical literature.

In 1572, just about the time of his marriage, Napier received title to the family estates. When time permitted from the daily running of his estates, John Napier played an active role in the Scottish Protestant reform movement. What time he had left he used to study mathematics. He is best known today for his invention of logarithms, but in his own time he was best known for his religious commentaries.

After he had published his logarithms, Napier published this small work on his Rabdologiae or, as they are better known, Napier's rods or Napier' bones. The devices were simple to use and quickly gained popularity. This work went through many different editions and was translated from the original Latin into all the major European languages. Examples of Napier's bones could be found, only a few years later, in such distant places as China and Japan. The basic concept of the bones was rapidly developed into a variety of forms ranging from inscribed circles and cylinders to metallic components in twentieth century calculating machines.

This work contains not only the description of the bones but also Napier's more sophisticated Multiplicationis promptuario and his binary-based chessboard calculation system.

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General notes on the condition of older books

Books as old as this usually suffer from some problems just because of the wear they have been subjected to over the many years of their existence. One usually noticeable condition item is known as *browning* or *foxing* of the paper - usually brown or yellow areas due to the chemical action of a micro-organism on the paper. This can vary dramatically from page to page, often depending on such variables as the contents of the paper used, the composition of the ink used by the printer, and the dampness (or lack of) that the work has been exposed to over the years. Where these images were badly foxed, some slight manipulation of the intensity of the colors has been done to ease the reading of the foxed page. Any other notable condition problem will be commented upon near the image concerned.

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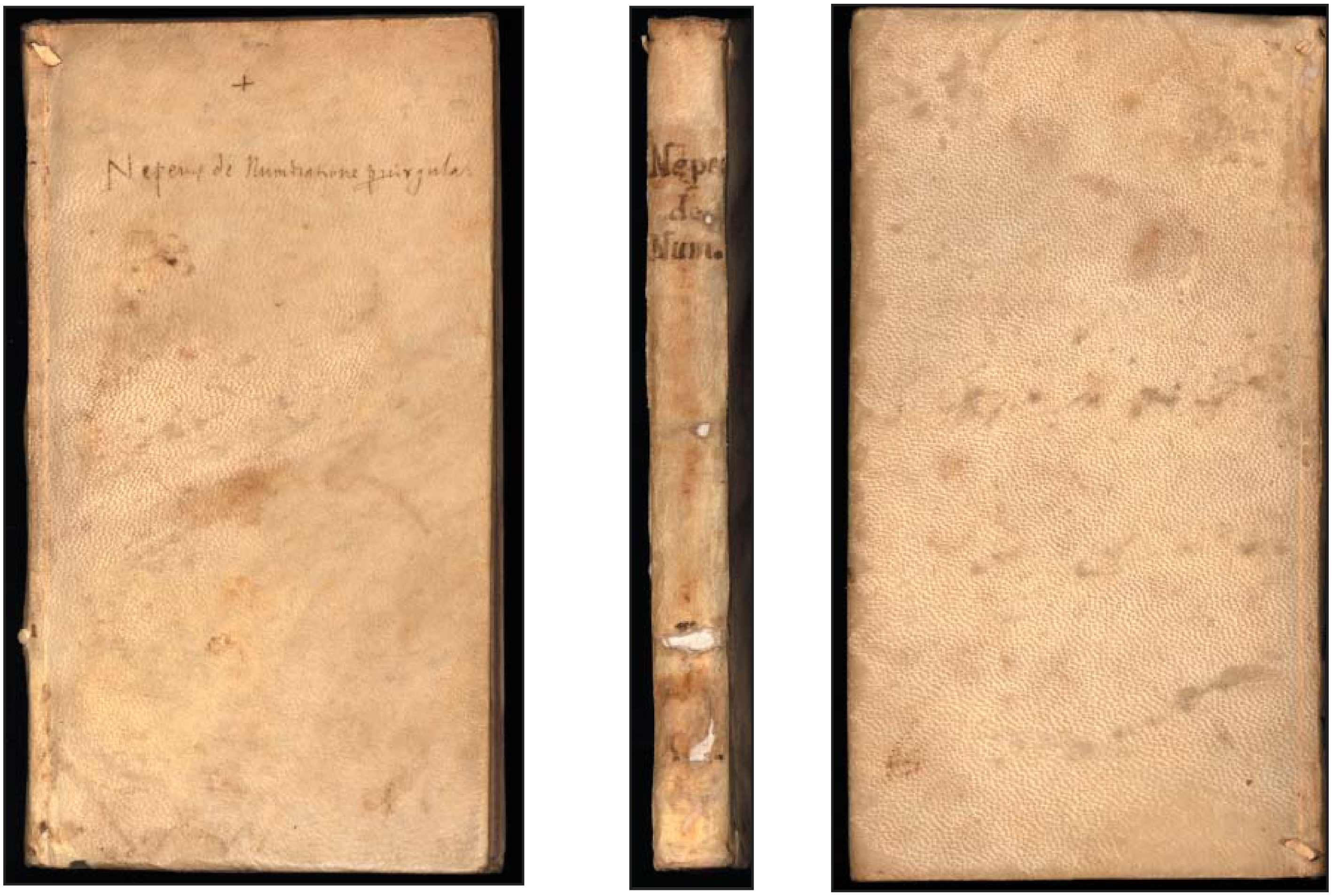
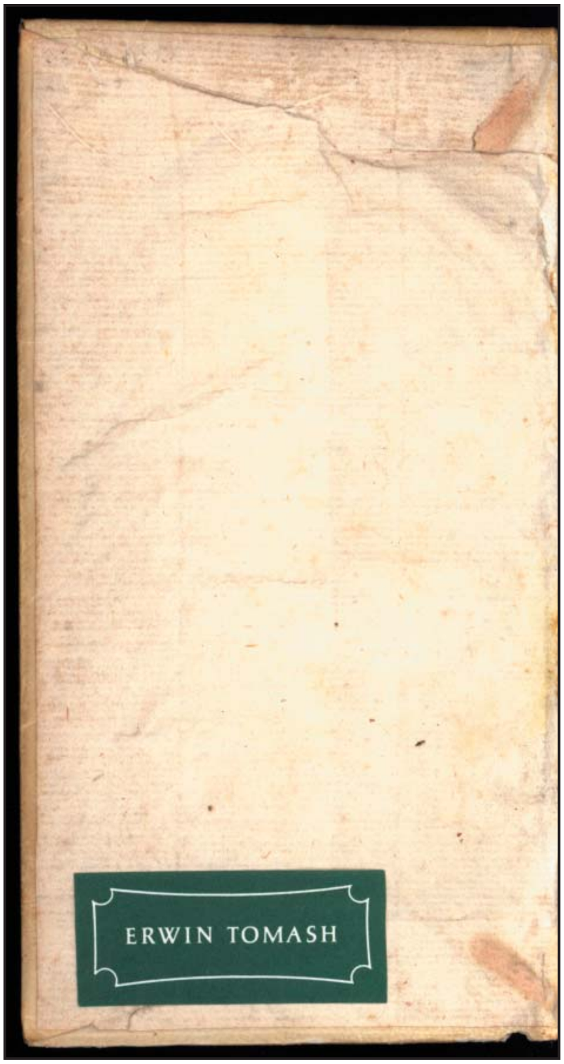
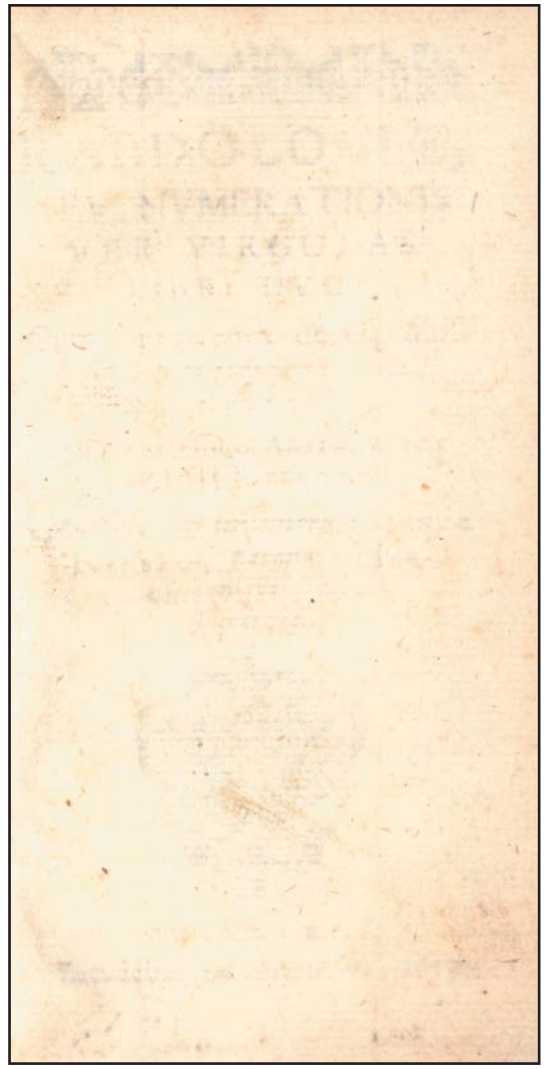
Napier, John (1550–1617) *Rabdologiae*, 1617, Edinburgh



Front cover, spine and rear cover





Front paste-down endpaper with the label of the Tomash Library.

Recto of the front free endpaper.

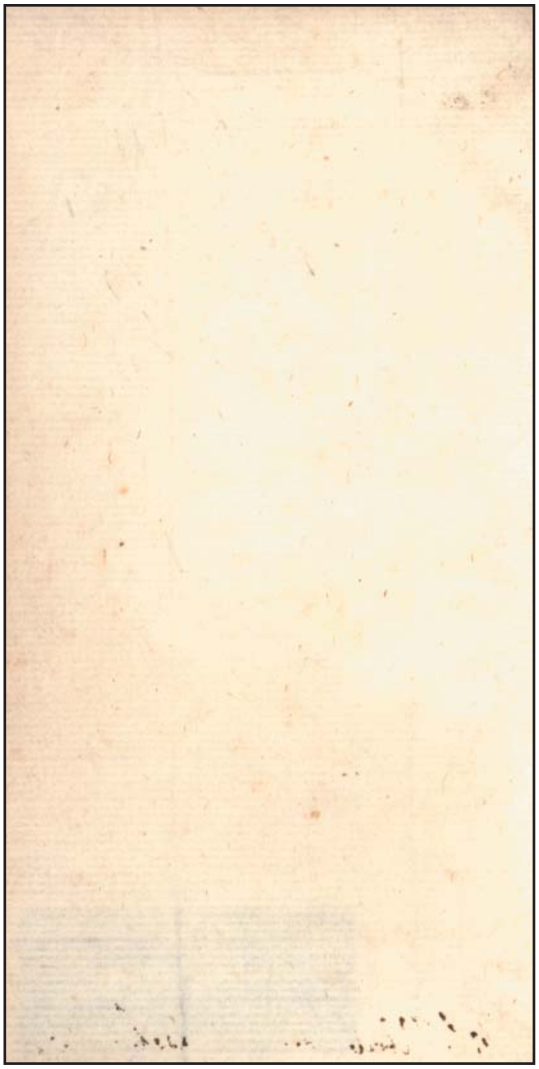
Much of the commentary on the following pages is based on several translated sources and on our own work. The major work used was

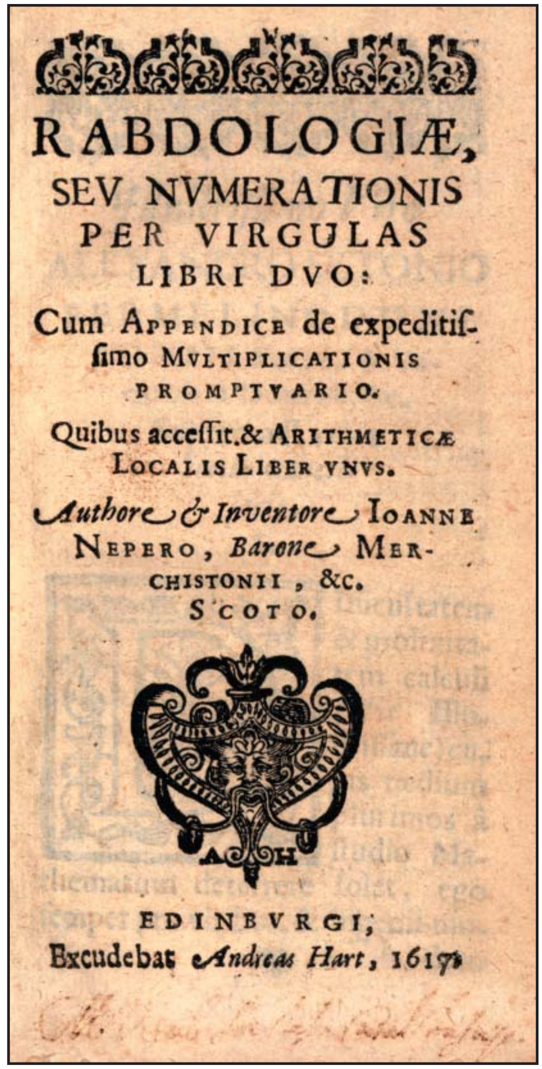
> Napier, John, *Rabdology*, Translated by William Frank Richardson, Charles Babbage Institute Reprint Series for the History Of Computing, Vol. 15. MIT Press (Cambridge, Mass., 1990).

To acknowledge each or the other sources would require another volume so we hope that the various authors will forgive our omissions in this regard.

From the Tomash Library on the History of Computing

Napier, John (1550–1617) *Rabdologiae*, 1617, Edinburgh





RABDOLOGIÆ,
SEV NVMERATIONIS
PER VIRGULAS
LIBRI DVO:
Cum Appendice de expeditissimo Mvltiplicationis
Promptvario.
Quibus accessit.& Arithmeticæ
Localis Liber vnvs.
Authore & Inventore Ioanne
Nepero, *Barone* Merchistonii, &c.
Scoto.

EDINBVRGI,
Excudebat *Andreas Hart*, 1617.

Verso of the front free endpaper.

Title page:

Rabdologiae, or the calculation with rods in two books.
With an appendix on a useful device for multiplication.
And one book on local arithmetic.
by the author and inventor, John Napier, Baron of Merchiston, a Scotsman.
Edinburgh
Published by Andrew Hart, 1617

From the Tomash Library on the History of Computing

Napier, John (1550–1617) *Rabdologiae*, 1617, Edinburgh

RABDOLOGIÆ,
SEV NVMERATIONIS
PER VIRGVLAS
LIBRI DVO:
Cum APPENDICE de expeditissimo MVLTIPLICATIONIS
PROMPTVARIO.
Quibus accessit & ARITHMETICÆ
LOCALIS LIBER VNVS.
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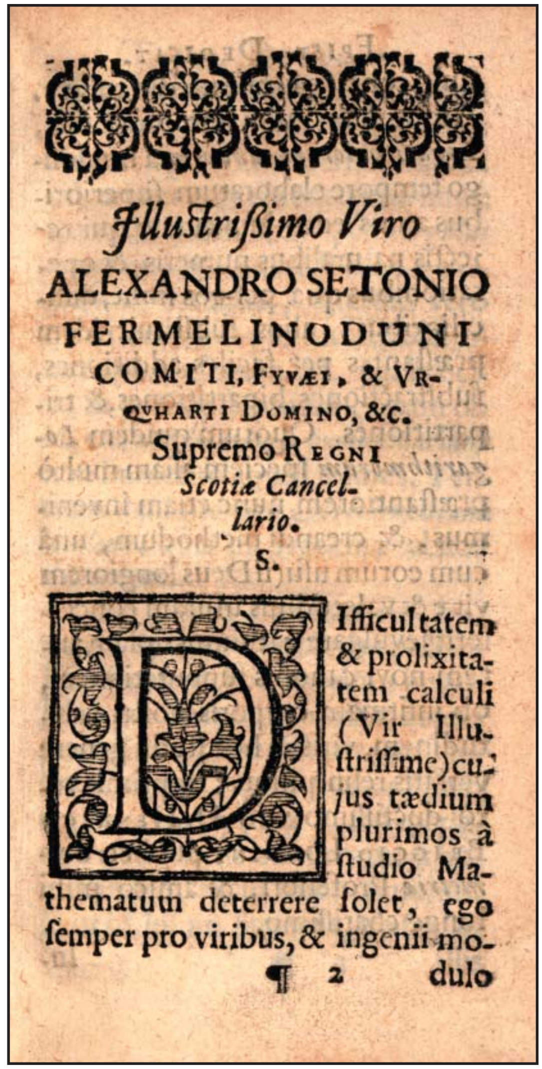

Illustrißimo Viro
ALEXANDRO SETONIO
FERMELINODUNI
COMITI, FYVÆI, & VR-
QVHARTI DOMINO, &c.
Supremo REGNI
Scotiæ Cancel-
lario.
S.

DIfficultatem & prolixitatem calculi (Vir Illustrissime) cujus tædium plurimos à studio Mathematum deterrere solet, ego semper pro viribus, & ingenii modulo

¶ 2 dulo

Verso of the title page

Napier dedicates this book to Alexander Sutton, the Chancellor of Scotland.

He indicates that he has recently published his book on logarithms. However he has now discovered a new type of logarithm (the base 10 logs) but because of ill health (he suffered from gout among other things) he is leaving the work of calculating them to his good friend Henry Briggs, a professor of geometry in London.

He has, at the urging of Sutton decided to publish this small book on three different methods of calculation. The first uses rods engraved with numbers, the second (his Promptuary of Multiplication) uses strips arranged in a box, and the third performs arithmetic on a chess board. He mentions that Sutton thought so much of the little rods that he had a set made. Rather than making them out of paper or wood, Sutton had them made them out of silver.

dulo conatus sum è medio tollere. Atque hoc mihi fine proposito, *Logarithmorum canonem* à me longo tempore elaboratum superioribus annis edendum curavi, qui rejectis naturalibus numeris, & operationibus quæ per eos fiunt, difficilioribus, alios substituit idem præstantes per faciles additiones, substractiones, bipartitiones, & tripartitiones. Quorum quidem *Logarithmorum* speciem aliam multò præstantiorem nunc etiam invenimus, & creandi methodum, unà cum eorum usu (si Deus longiorem vitæ & valetudinis usuram concesserit) evulgare statuimus: ipsam autem novi canonis supputationem, ob infirmam corporis nostri valetudinem, viris in hoc studii genere versatis relinquimus: imprimis verò doctissimo viro D. HENRICO BRIGGIO LONDINI publico *Geometriæ* Professori, & amico mihi longè charissimo.

Intereà tamen in gratiam eorum qui per ipsos numeros naturales oblatos operari maluerint, tria alia calculi compendia excogitavimus: quorum primum est per *virgulas numeratrices*, quod RABDOLOGIAM vocamus: alterum verò quod omnium pro multiplicatione expeditissimum est, per lamellas in pyxide dispositas, quam ob id, *Multiplicationis promptuarium* non immeritò appellabimus. Tertium denique per *Arithmeticam localem*, quæ in Scacchiæ abaco exercetur.

Ut autem libellum de FABRICA & VSV *virgularum* publici juris facerem, hoc imprimis impulit, quod eas non solum viderem permultis ita placuisse, ut jam ferè sint vulgares, & in exteras etiam regiones deferantur: sed perlatum quoque sit ad aures meas humanitatem tuam mihi consuluisse ut id ipsum facerem, ne forsan illis alieno nomine editis, cum *Virgilio* canere cogerer,

Hos ego versiculos feci, &c.

Atque hoc tuæ amplitudinis amantissimum consilium apud me maximum pondus habere debuit: & certè sine eo vix unquam hoc de virgulis opusculum (cui reliqua duo adjunximus compendia) in lucem prodiisset.

Si quæ igitur gratiæ à *Mathematum* cultoribus ob hos libellos debentur, eas omnes (tu Vir Clarissime) tuo tibi jure vendicas, ad quem non modò ut patronum, sed potius ut alterum parentem liberè transvolant: præsertim quum exploratum habeam te meas illas virgulas tanti fecisse, ut non ex vulgari materia, sed ex argento fieri curaveris.

Accipe igitur æquo animo (Vir Illustrissime) hoc opusculum qualecun-

lecunque: ejusque licet tanto Mæcenate indigni, ut tui tamen fœtus patrocinium suscipe: Sicut & te Iustitiæ æquitatisque patronum diu nobis & Reipublicæ incolumem servari enixè à DEO optamus.

Amplitudini tuæ meritò addictissimus

IOANNES NEPERUS MERCHISTONII BARO.

From the Tomash Library on the History of Computing

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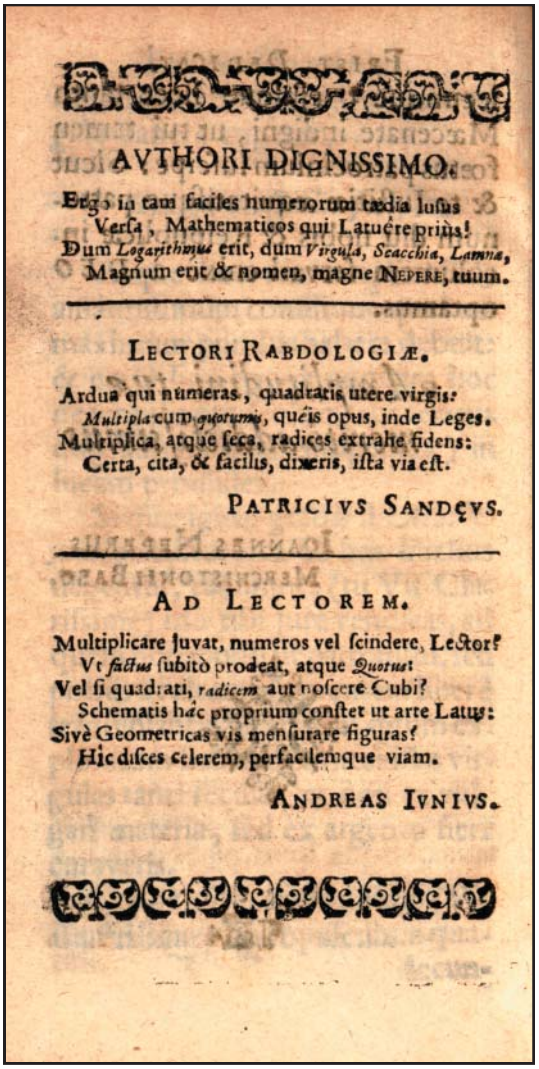


AVTHORI DIGNISSIMO.

Ergo in tam faciles numerorum tædia lusus
Versa, Mathematicos qui Latuêre priùs!
Dum *Logarithmus* erit, dum *Virgula, Scacchia, Lamna,*
Magnum erit & nomen, magne NEPERE, tuum.

LECTORI RABDOLOGIÆ.

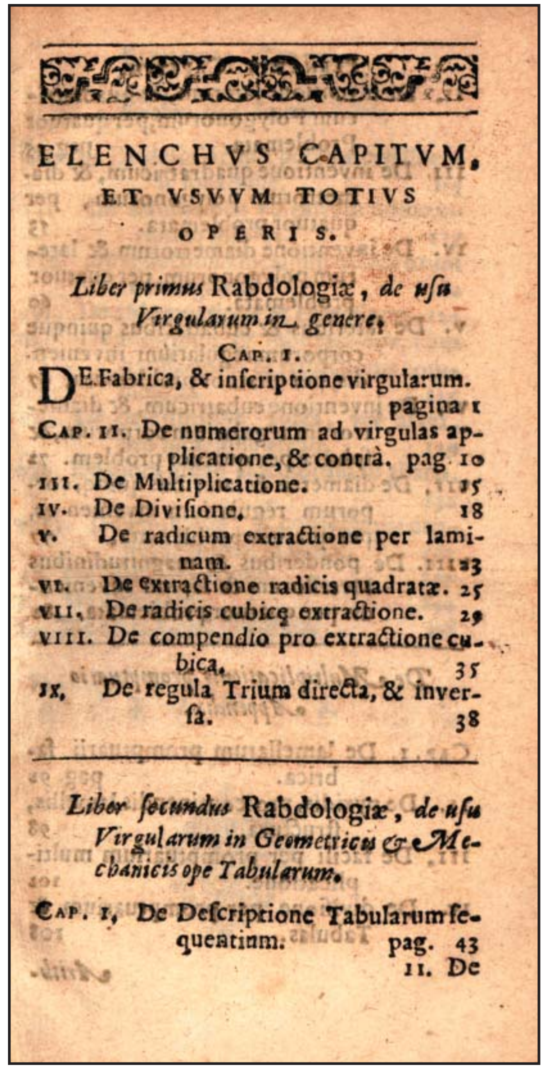
Ardua qui numeras, quadratis utere virgis:
Multipla cum *quotumis*, quêis opus, inde Leges.
Multiplica, atque seca, radices extrahe fidens:
Certa, cita, & facilis, dixeris, ista via est.

PATRICIVS SANDÆVS.

AD LECTOREM.

Multiplicare juvat, numeros vel scindere, Lector?
Vt *factus* subitò prodeat, atque *Quotus*:
Vel si quadrati, *radicem* aut noscere Cubi?
Schematis hâc proprium constet ut arte Latus:
Sivè Geometricas vis mensurare figuras?
Hic disces celerem, perfacilemque viam.

ANDREAS IVNIVS.



ELENCHVS CAPITVM, ET VSVVM TOTIVS OPERIS.

Liber primus Rabdologiæ, *de usu Virgularum in genere.*

CAP. I.

Liber secundus Rabdologiæ, *de usu Virgularum in Geometricis & Mechanicis ope Tabularum.*

It was common in books of this era to include a poem or two about the author or the subject matter. The first of these lauds Napier for the inventions, the second (by Patrick Sandys) mentions the rods and the third (by Andrew Young) indicates that the methods (the rods) are accurate, quick, and useful. Andrew Young was a Professor of Philosophy at the University of Edinburgh and, in 1620, he was also appointed as the first Professor of Mathematics at that same institution. Both authors also wrote poems for Napier's book on logarithms (see the file for Napier's *Descriptio*, 1614).

Here follows the Table of Contents:

From the Tomash Library on the History of Computing

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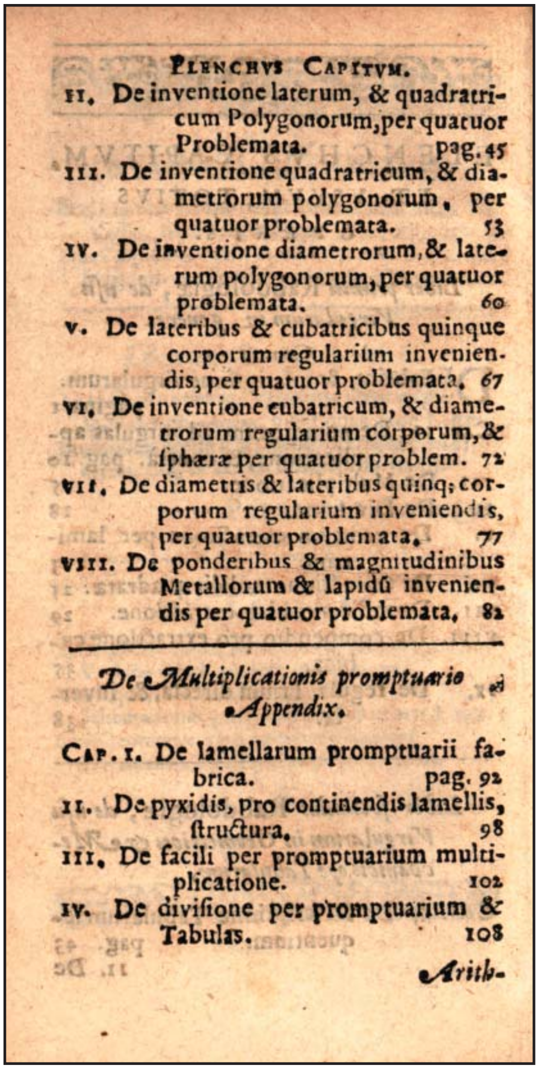

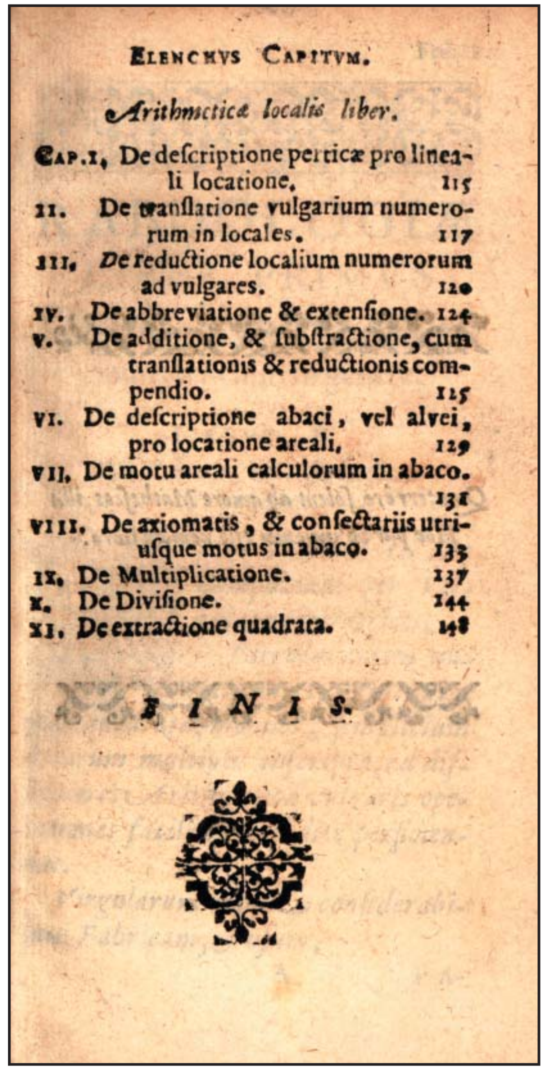

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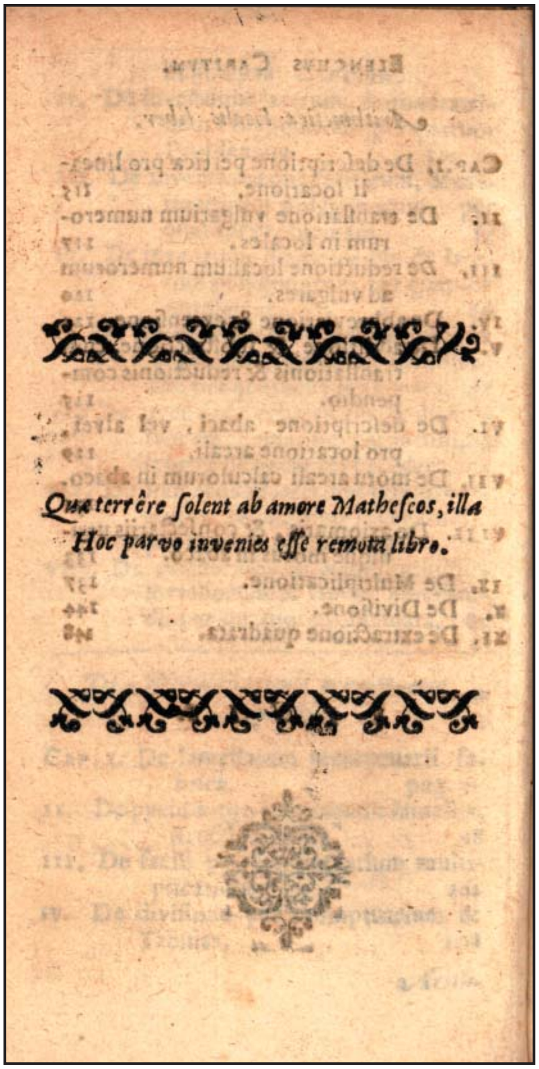
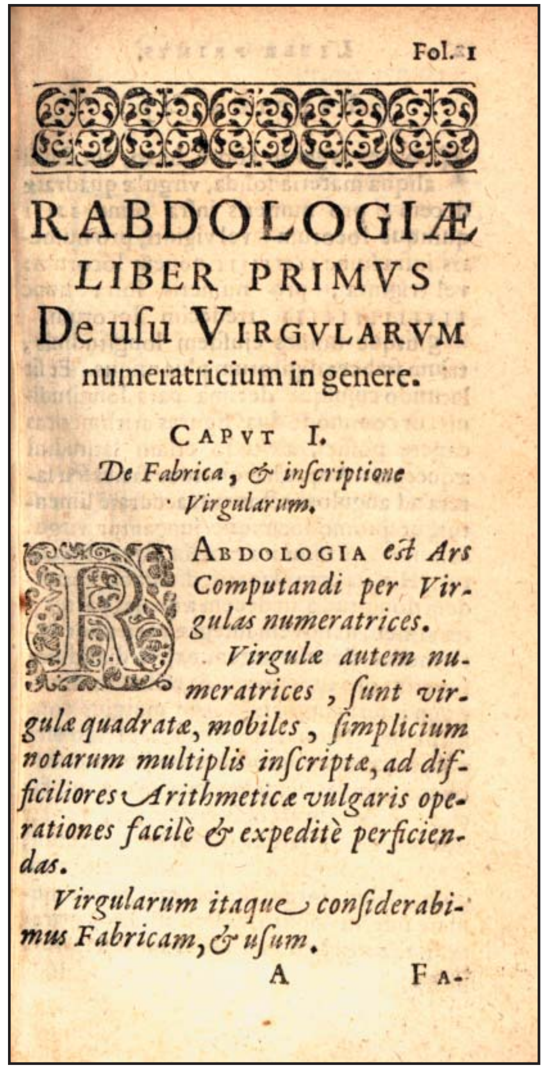


Quæ terrêre solent ab amore Mathescos, illa
Hoc parvo invenies esse remota libro.



Fol. 1

RABDOLOGIÆ
LIBER PRIMVS
De usu VIRGVLARVM
numeratricium in genere.

CAPVT I.
De Fabrica, & inscriptione Virgularum.

RABDOLOGIA *est Ars Computandi per Virgulas numeratrices.*

Virgulæ autem numeratrices, sunt virgulæ quadratæ, mobiles, simplicium notarum multiplis inscriptæ, ad difficiliores Arithmeticæ vulgaris operationes facilè & expeditè perficiendas.

Virgularum itaque considerabimus Fabricam, & usum.

A FA-

A two-line poem that suggests this book will remove any difficulties that beginners have with arithmetic operations.

Book One
The use of the rods.

Chapter I: Their construction and inscription.

From the Tomash Library on the History of Computing

Napier, John (1550–1617) *Rabdologiae*, 1617, Edinburgh

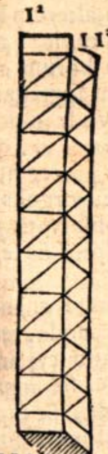
FABRICA *sic fit.*

FIANT ex argento, ebore, buxo, aut simili aliqua materia solida, virgulæ quadratæ decem, pro numeris infra hunc IIIII quinque locorum; vel viginti, pro numeris infra hunc IIIIIIIII nouem locorum: vel triginta, pro numeris infra hunc IIIIIIIIIIIII tredecim locorum.

Sintque omnes ejusdem longitudinis, trium scilicet digitorum plus minus. Et sit latitudo cujusque decima pars longitudinis, ut commodè duas figuras arithmeticas capere possit, altitudo etiam latitudini æquetur. Atque hæ quatuor facies seu latera ad angulos rectos tam accuratè limentur, ut quomodocunque jungantur virgulæ, omnes quasi unica tabella plana videantur. His ita complanatis, dividatur earundem longitudo in decem æquales partes: ita tamen, ut novem integræ partes sint intermediæ, decimæ autem partis dimidium superius pro superiore, & reliquum dimidium inferius pro inferiore margine constituatur. Proinde per singula divisionum puncta, ducantur rectæ lineæ, quæ distribuant singulas singularum virgularum facies, in novem areolas quadratas, præter margines: quarum quælibet bisecetur, ductis diagoniis, à sinistro & inferiore angulo, ad superiorem & dextrum, ut in schemate inferiùs posito, videre est. Et ita paratæ sunt virgulæ ad numerorum inscriptionem,

Pri-

Schema Virgulæ.



Primò itaque positis ob oculos virgulis, signentur (memoriæ & doctrinæ gratia) earundem facies I^{a}, II^{a}, III^{a}, & IIII^{a}, notis delebilibus his aut aliis; ut prima facies dicatur, quæ nunc ob oculos ponitur, secunda, quæ dextram spectat, tertia, quæ terram, & quarta, quæ lævam.

Secundò observandum est, quod prima figura quæ in capite seu prima areola cujusque faciei est ponenda, & in dextra parte areolæ sculpenda, simplex figura est, & simplum dicitur: quæ in secunda areola sequuntur, sunt ejusdem figuræ duplum: quæ in tertia areola triplum, quæ in quarta quadruplum, & sic de reliquis multiplis usque ad noncuplum inclusivè: quorum si quod unicâ tantum figurâ constet, illa est in dextra parte suæ areolæ sculpenda: si vero duabus, dextra dextrorsum, & læva lævorsum in areola scribatur.

Tertiò notandum est, quod cujusque

 virgu-

Their construction

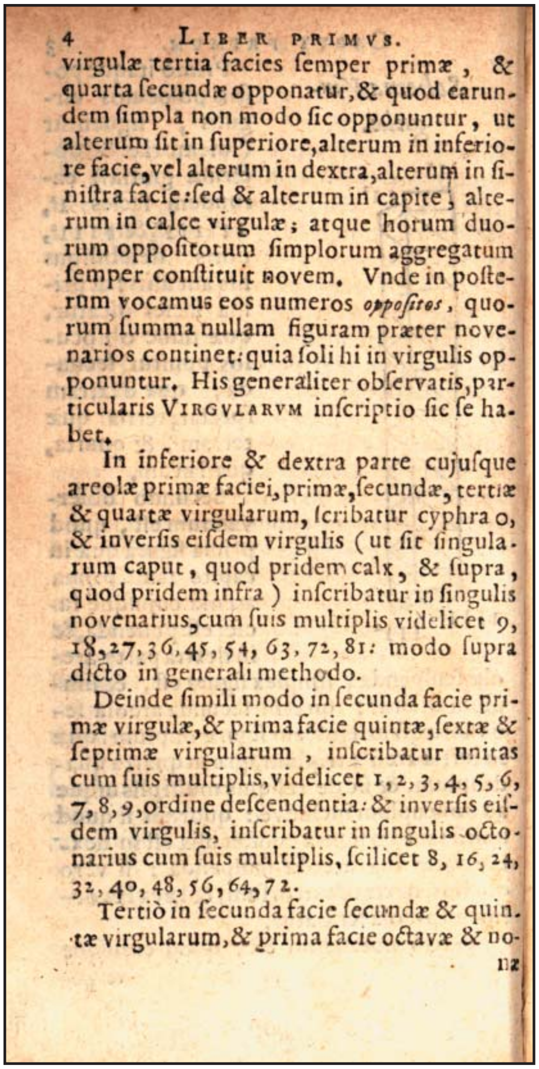
Make some square rods from silver, wood or other material. Make 10 rods for numbers less than 11,111; 20 rods for numbers less than 111,111,111; 30 rods for numbers less than 1,111,111,111,111; etc. They should all be of equal length (Napier suggests the breadth of three fingers) and about $^{1}/_{10}$ as wide as they are long - enough to easily mark down two single digit numbers.

Mark them on all four faces as shown in the figure - 9 squares with diagonal lines and a smaller space (one half the size of a square) at each end. Napier marks the four faces of each rod as I, II, III and IIII.

Each face should be marked as follows:

In the first square at the top, put down a digit in the lower triangle (called a *simple digit*).

On each square below write down the 2ed, 3ed, ... 9th multiple of that digit (units digit in the lower triangle and tens digit in the upper triangle of each square)



4 LIBER PRIMVS.

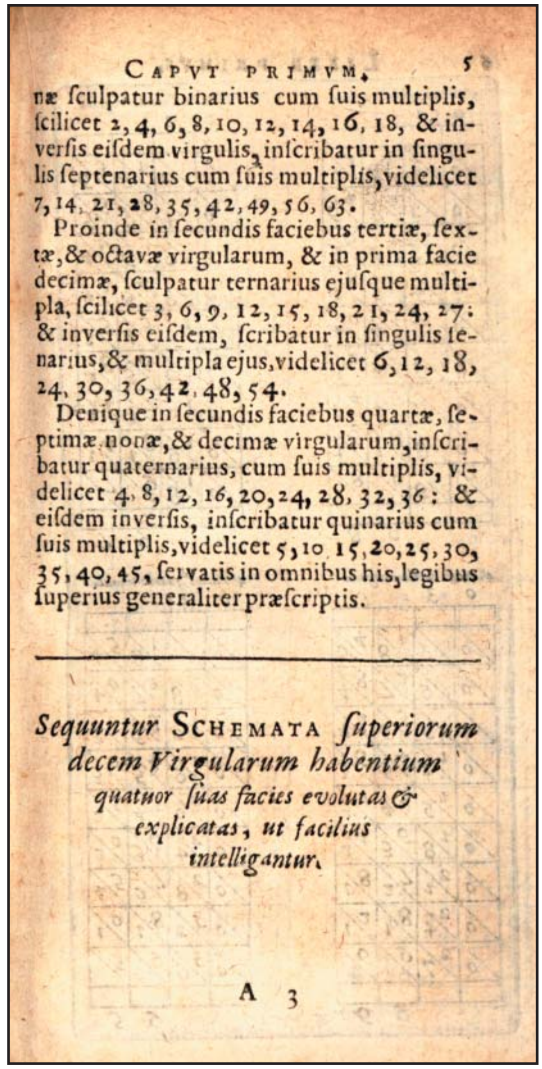
virgulæ tertia facies ſemper primæ, & quarta ſecundæ opponatur, & quod earundem ſimpla non modo ſic opponuntur, ut alterum ſit in ſuperiore, alterum in inferiore facie, vel alterum in dextra, alterum in ſiniſtra facie: ſed & alterum in capite, alterum in calce virgulæ; atque horum duorum oppoſitorum ſimplorum aggregatum ſemper conſtituit novem. Vnde in poſterum vocamus eos numeros *oppoſitos*, quorum ſumma nullam figuram præter novenarios continet: quia ſoli hi in virgulis opponuntur. His generaliter obſervatis, particularis VIRGVLARVM inſcriptio ſic ſe habet.

In inferiore & dextra parte cujuſque areolæ primæ faciei, primæ, ſecundæ, tertiæ & quartæ virgularum, ſcribatur cyphra o, & inverſis eiſdem virgulis (ut ſit ſingularum caput, quod pridem calx, & ſupra, quod pridem infra) inſcribatur in ſingulis novenarius, cum ſuis multiplis videlicet 9, 18, 27, 36, 45, 54, 63, 72, 81: modo ſupra dicto in generali methodo.

Deinde ſimili modo in ſecunda facie primæ virgulæ, & prima facie quintæ, ſextæ & ſeptimæ virgularum, inſcribatur unitas cum ſuis multiplis, videlicet 1, 2, 3, 4, 5, 6, 7, 8, 9, ordine deſcendentia: & inverſis eiſdem virgulis, inſcribatur in ſingulis octonarius cum ſuis multiplis, ſcilicet 8, 16, 24, 32, 40, 48, 56, 64, 72.

Tertiò in ſecunda facie ſecundæ & quintæ virgularum, & prima facie octavæ & no-

næ



CAPVT PRIMVM. 5

næ ſculpatur binarius cum ſuis multiplis, ſcilicet 2, 4, 6, 8, 10, 12, 14, 16, 18, & inverſis eiſdem virgulis, inſcribatur in ſingulis ſeptenarius cum ſuis multiplis, videlicet 7, 14, 21, 28, 35, 42, 49, 56, 63.

Proinde in ſecundis faciebus tertiæ, ſextæ, & octavæ virgularum, & in prima facie decimæ, ſculpatur ternarius ejuſque multipla, ſcilicet 3, 6, 9, 12, 15, 18, 21, 24, 27: & inverſis eiſdem, ſcribatur in ſingulis ſenarius, & multipla ejus, videlicet 6, 12, 18, 24, 30, 36, 42, 48, 54.

Denique in ſecundis faciebus quartæ, ſeptimæ, nonæ, & decimæ virgularum, inſcribatur quaternarius, cum ſuis multiplis, videlicet 4, 8, 12, 16, 20, 24, 28, 32, 36: & eiſdem inverſis, inſcribatur quinarius cum ſuis multiplis, videlicet 5, 10, 15, 20, 25, 30, 35, 40, 45, ſervatis in omnibus his, legibus ſuperius generaliter præſcriptis.

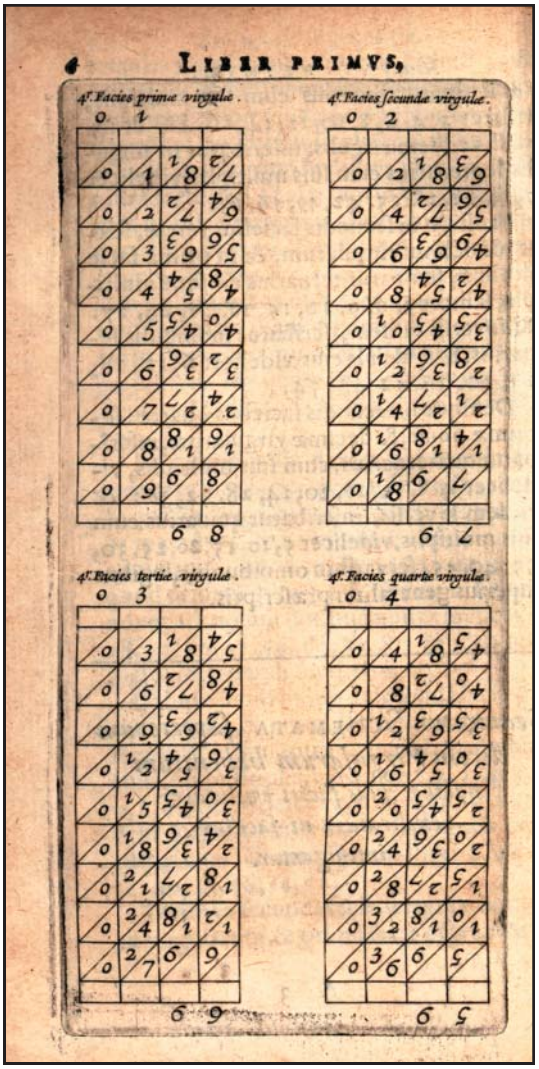
Sequuntur SCHEMATA *ſuperiorum decem Virgularum habentium quatuor ſuas facies evolutas & explicatas, ut facilius intelligantur.*

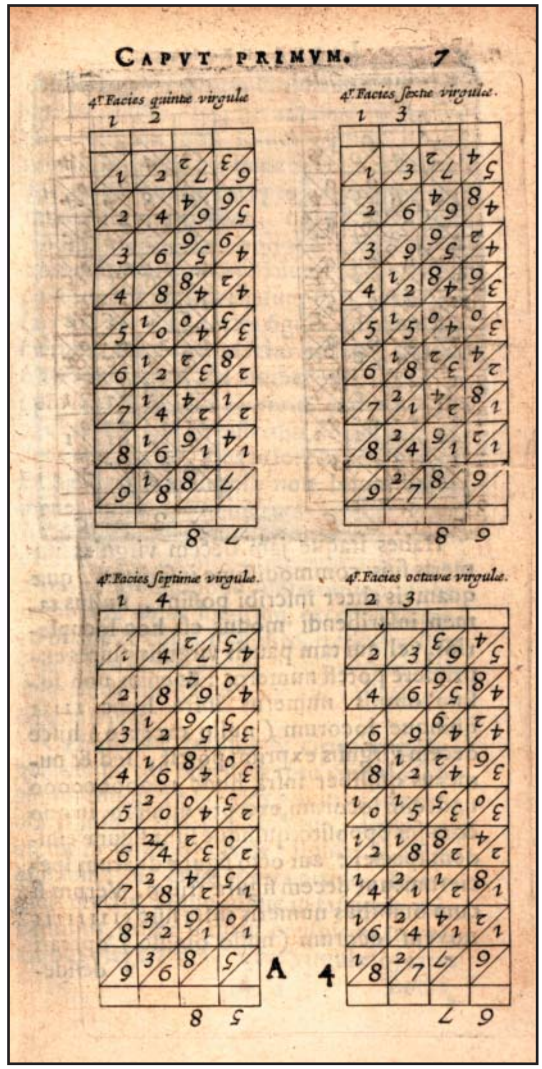
A 3

Note, from the diagrams that follow, that the faces are marked with different *simple digits* so that the *simple digits* on opposite faces add up to 9 (e.g., 1 and 8, 0 and 9, 2 and 7, etc.). Also the opposite faces are marked so that the top of face I is actually the bottom of face III and similarly for face II and IIII. (While this 180° rotation is not strictly necessary, it make the use of the rods more convenient.) Napier then goes into detail about what numbers should appear on each face of each rod, but this is obvious from the diagrams that follow.

From the Tomash Library on the History of Computing

Napier, John (1550–1617) *Rabdologiae*, 1617, Edinburgh





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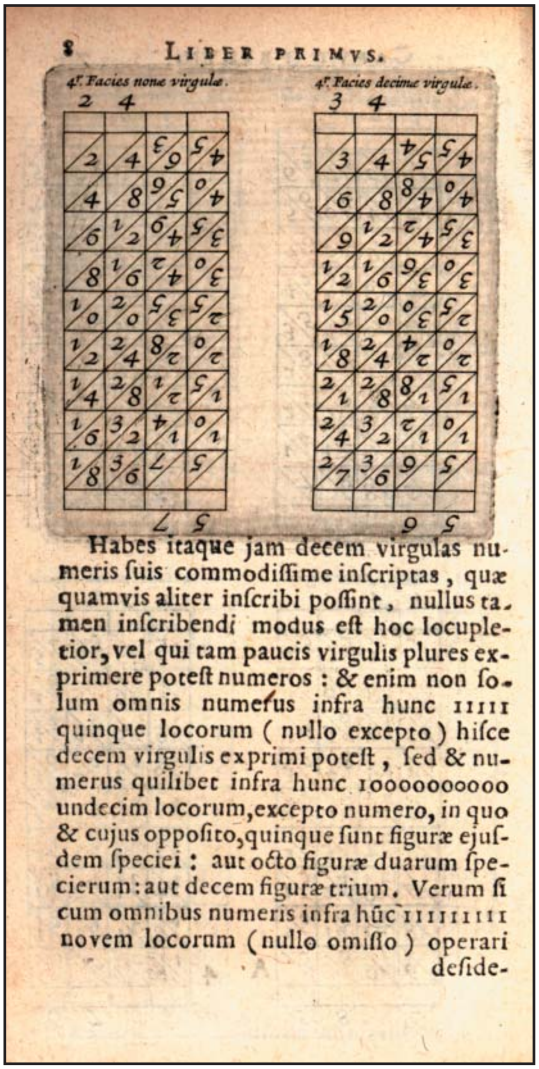
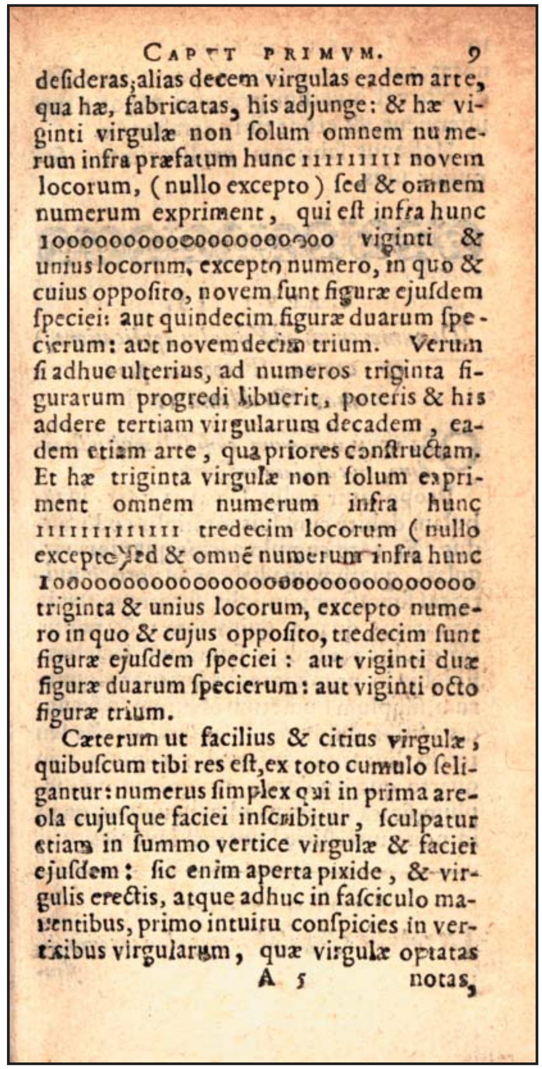


8 LIBER PRIMVS.

Habes itaque jam decem virgulas numeris ſuis commodiſſime inſcriptas, quæ quamvis aliter inſcribi poſſint, nullus tamen inſcribendi modus eſt hoc locupletior, vel qui tam paucis virgulis plures exprimere poteſt numeros: & enim non ſolum omnis numerus infra hunc 11111 quinque locorum (nullo excepto) hiſce decem virgulis exprimi poteſt, ſed & numerus quilibet infra hunc 10000000000 undecim locorum, excepto numero, in quo & cujus oppoſito, quinque ſunt figuræ ejuſdem ſpeciei: aut octo figuræ duarum ſpecierum: aut decem figuræ trium. Verum ſi cum omnibus numeris infra hũc 111111111 novem locorum (nullo omiſſo) operari deſide-



CAPVT PRIMVM. 9

deſideras; alias decem virgulas eadem arte, qua hæ, fabricatas, his adjunge: & hæ viginti virgulæ non ſolum omnem numerum infra præfatum hunc 111111111 novem locorum, (nullo excepto) ſed & omnem numerum exprimẽt, qui eſt infra hunc 100000000000000000000 viginti & unius locorum, excepto numero, in quo & cuius oppoſito, novem ſunt figuræ ejuſdem ſpeciei: aut quindecim figuræ duarum ſpecierum: aut novemdecim trium. Verum ſi adhuc ulterius, ad numeros triginta figurarum progredi libuerit, poteris & his addere tertiam virgularum decadem, eadem etiam arte, qua priores conſtructam. Et hæ triginta virgulæ non ſolum expriment omnem numerum infra hunc 1111111111111 tredecim locorum (nullo excepto) ſed & omnẽ numerum infra hunc 1000000000000000000000000000000 triginta & unius locorum, excepto numero in quo & cujus oppoſito, tredecim ſunt figuræ ejuſdem ſpeciei: aut viginti duæ figuræ duarum ſpecierum: aut viginti octo figuræ trium.

Cæterum ut facilius & citius virgulæ, quibuſcum tibi res eſt, ex toto cumulo ſeligantur: numerus ſimplex qui in prima areola cujuſque faciei inſcribitur, ſculpatur etiam in ſummo vertice virgulæ & faciei ejuſdem: ſic enim aperta pixide, & virgulis erectis, atque adhuc in faſciculo manentibus, primo intuitu conſpicies in verticibus virgularum, quæ virgulæ optatas

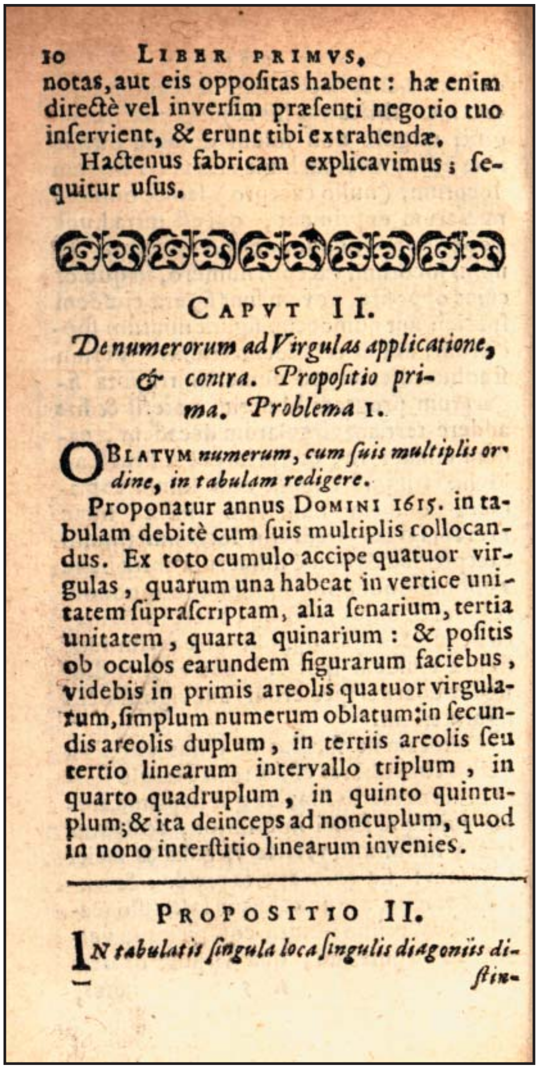
A 5 notas,

Having made the rods as described, you will now have 10 rods that can be used for calculating with numbers less than 11,111. They can also be used for calculating with numbers less than 10,000,000,000 when a single digit occurs 5 times or any two digits occur 8 times or any three digits occur 10 times - in which case you will have to make another 10 rods to deal with these exceptions. Similar statements are made about the limitations of a set of 20 rods and of 30 rods.

The very top of each rod should be marked with the simple number that is to be found on the side faces. This is so that, when the rods are all together in a bunch (as standing vertically in their box) you may easily see which ones need to be picked out to do your calculation. These top numbers are shown in the diagrams.

From the Tomash Library on the History of Computing

Napier, John (1550–1617) *Rabdologiae*, 1617, Edinburgh



10 LIBER PRIMVS,

notas, aut eis oppoſitas habent: hæ enim directè vel inverſim præſenti negotio tuo inſervient, & erunt tibi extrahendæ.

Hactenus fabricam explicavimus; ſequitur uſus,

CAPVT II.

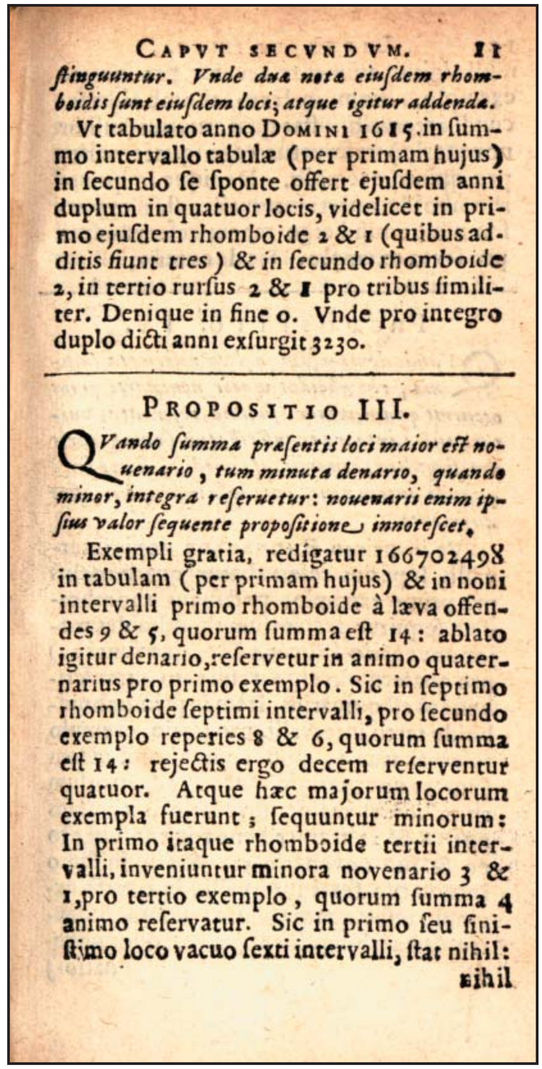
De numerorum ad Virgulas applicatione, & contra. Propoſitio prima. Problema 1.

OBLATVM *numerum, cum ſuis multiplis ordine, in tabulam redigere.*

Proponatur annus DOMINI 1615. in tabulam debitè cum ſuis multiplis collocandus. Ex toto cumulo accipe quatuor virgulas, quarum una habeat in vertice unitatem ſupraſcriptam, alia ſenarium, tertia unitatem, quarta quinarium: & poſitis ob oculos earundem figurarum faciebus, videbis in primis areolis quatuor virgularum, ſimplum numerum oblatum; in ſecundis areolis duplum, in tertiis areolis ſeu tertio linearum intervallo triplum, in quarto quadruplum, in quinto quintuplum; & ita deinceps ad noncuplum, quod in nono interſtitio linearum invenies.

PROPOSITIO II.

I*N tabulatis ſingula loca ſingulis diagoniis diſtin-*



CAPVT SECVNDVM. 11

ſtinguuntur. Vnde duæ notæ eiuſdem rhomboidis ſunt eiuſdem loci; atque igitur addendæ.

Vt tabulato anno DOMINI 1615. in ſummo intervallo tabulæ (per primam hujus) in ſecundo ſe ſponte offert ejuſdem anni duplum in quatuor locis, videlicet in primo ejuſdem rhomboide 2 & 1 (quibus additis fiunt tres) & in ſecundo rhomboide 2, in tertio rurſus 2 & 1 pro tribus ſimiliter. Denique in fine 0. Vnde pro integro duplo dicti anni exſurgit 3230.

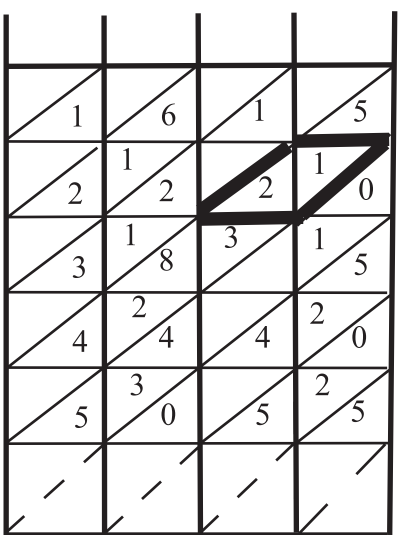
PROPOSITIO III.

Q*Vando ſumma præſentis loci maior eſt novenario, tum minuta denario, quando minor, integra reſervetur: novenarii enim ipſius valor ſequente propoſitione innoteſcet.*

Exempli gratia, redigatur 166702498 in tabulam (per primam hujus) & in noni intervalli primo rhomboide à læva offendes 9 & 5, quorum ſumma eſt 14: ablato igitur denario, reſervetur in animo quaternarius pro primo exemplo. Sic in ſeptimo rhomboide ſeptimi intervalli, pro ſecundo exemplo reperies 8 & 6, quorum ſumma eſt 14: rejectis ergo decem reſerventur quatuor. Atque hæc majorum locorum exempla fuerunt; ſequuntur minorum: In primo itaque rhomboide tertii intervalli, inveniuntur minora novenario 3 & 1, pro tertio exemplo, quorum ſumma 4 animo reſervatur. Sic in primo ſeu finiſtimo loco vacuo ſexti intervalli, ſtat nihil: nihil

Chapter II: Use of the rods.
Setting up the rods on a table to show a number.

Take, for example, the year 1615. Select rods with the numbers 1, 6, 1 and 5 on their faces and place them in order on the table. They will now show, in the top square, the number 1615 and in the second square they will show twice that number, and in the ninth square they will show the nines multiple of the number 1615.



With the rod set up to represent a number, you will observe that parallelograms are formed in each row with two halves of each parallelogram on adjacent rods - the diagonal lines being the right and left sides of each parallelogram. Napier does not provide a diagram of this situation, so readers might find it easier to examine the situation in the adjacent diagram. Reading, on the second line, from right to left: there is a lower triangle containing a “0,” then a parallelogram (shown in heavy black lines just to be sure you understand that the figure spans two of Napier’s rods) containing a “2” and “1,” another parallelogram containing only a “2,” a final parallelogram containing another “2” and a “1,” and finally and upper triangle containing a blank (“0”). Adding together the single digits in each parallelogram (2 + 1 = 3) the product of 1615*2 can be read as being 3230.

nihil igitur animo reſervetur pro quarto exemplo. Item in ſexto rhomboide ſecundi intervalli, ſtat quaternarius(minor novenario) reſervetur ergo quaternarius pro quinto exemplo. Denique in quinto rhomboide quarti intervalli, ſtat cyphra ſeu nihil: nihil ergo manet reſervandum pro ſexto exemplo. Sic de cæteris.

PROPOSITIO. IV.

Qvando dextrorſum à loco reſeruatæ ſummæ, rhombeides maior nouenario prius occurrit quam minor; ſumma reſeruata, vnitate aucta, integra tranſcribatur, (cum cyphris pro ſingulis novenariis intermediis, ſi qui ſint) quando autem non, cum ipſis nouenariis, ſine augmento.

Vt in primo RHOMBOIDE noni intervalli, ſuperius reſervabatur quaternarius pro primo exemplo, poſt cujus rhomboidem ſequitur dextrorſum rhomboides major novenario, ſcilicet 4 & 6, (quæ ſunt 10:) pro quaternario ergo reſervato, tranſcribendus eſt quinarius. Item in primo rhomboide tertii intervalli, pro tertio exemplo reſervabatur 4, poſt cuius rhomboidem (præter novenarium) ſuccedit dextrorſum rhomboides maior novenario, conſtans notis 8 & 2: tranſcribenda igitur ſunt pro eo & ſuo novenario 50. Sic in ſiniſtimo loco vacuo ſexti intervalli, in quarto exemplo reſervabatur nihil in animo: & dextrorſum ab hujus loco, (præter duos novenarios)

narios) ſucceſſit rhomboides novenario major, 6 & 4 ſeu 10: augendum ergo erit nihilum vnitate, & pro nihilo cum duabus cyphris, tranſcribenda erunt 100. Item ſuperius in ſeptimo rhomboide ſeptimi intervalli, reſervabatur quaternarius pro ſecundo exemplo, quem ſequebatur immediatè dextrorſum rhomboides non maior novenario, ſcilicet 3 & 5, quæ ſunt 8: integer ergo quaternarius eſt ſine augmento tranſcribendus. Item in ſexto rhomboide ſecundi intervalli reſervatus eſt ſuperius in quinto exemplo quaternarius, & hunc rhomboidem ſequebatur (præter duos novenarios) ſenarius(novenario ſcilicet haud maior:) integer ergo & ſine augmento tranſcribatur quaternarius cum ſuis binis novenariis ſic, 499. Denique in quinto rhomboide quarti intervalli, reſervabatur nihil pro ſexto exemplo, cuius rhomboidem ſequebatur (præter tres novenarios) binarius, qui (cum novenarium non excedat) reſervatum nihilum cum ſuis tribus novenariis abſque augmento ſcribenda eſſe arguit, hoc modo, 0999, & ſic de reliquis.

PROPOS. V. PROBL. II.

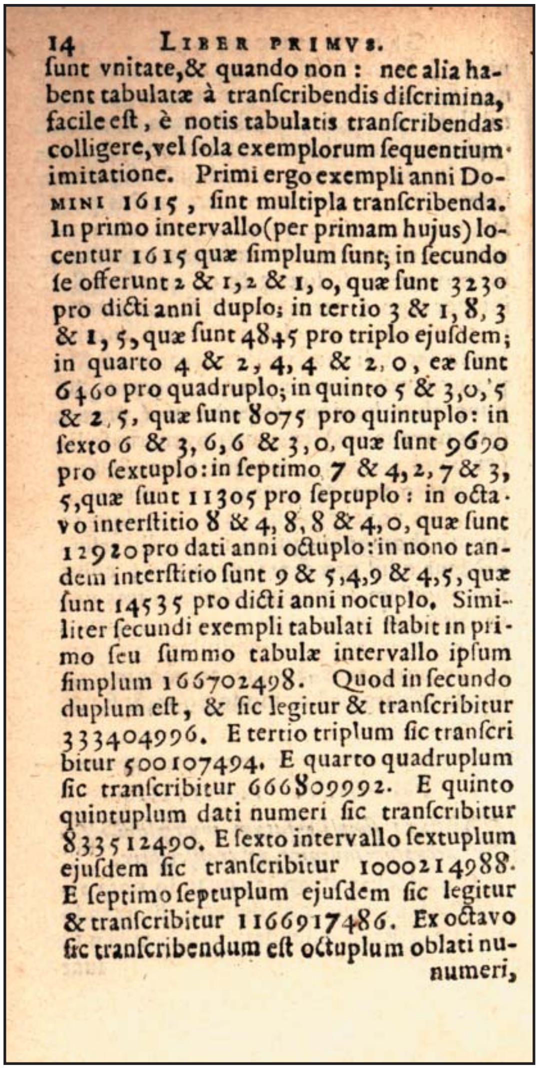
OBLATI *ſimpli optatum multiplum infra decuplum inuenire, & tranſcribere.*

Cum doceat Prop. 3. quando notæ tabulatæ ſunt minuendæ denario, & quando non: & 4. Prop. oſtendat quando augendæ ſunt

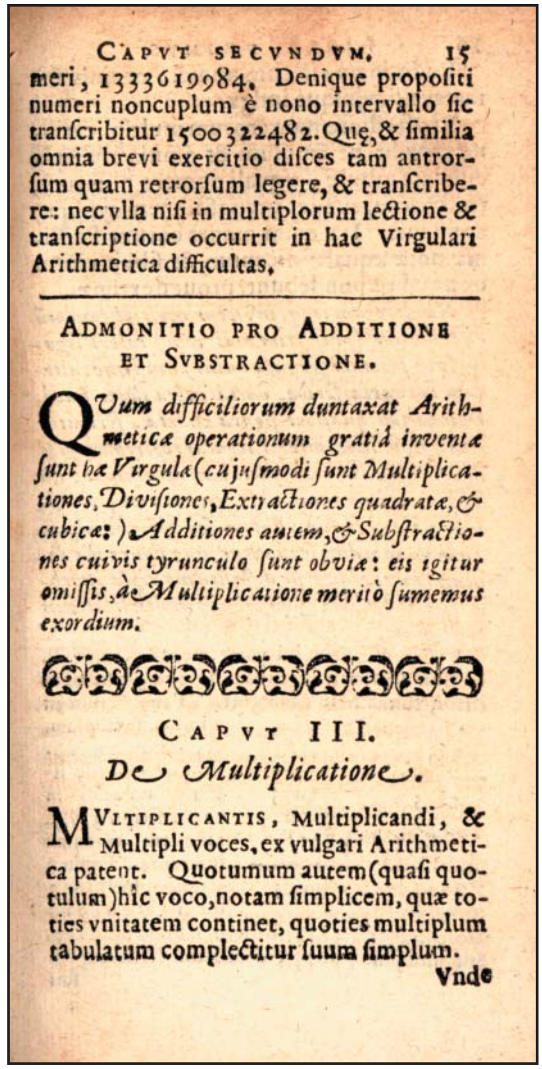
Similarly, by reading the fifth line, the product of 1625*5 is (5+3)(0)(5+2)(5) or 8075.

Proposition 3: when the total in any parallelogram is greater than 10, carry the tens digit to the next left position and add it to the figures found there. Proposition IV: As in ordinary arithmetic, if the contents of any parallelogram adds up to 10 or more, the tens digit must be carried over to the next position to the left and if this causes that parallelogram to be 10 or greater then the tens digit from that last addition must be carried to the left again, etc.

Proposition 5 simply provides a few more examples of how to add up numbers across the rods.



14 LIBER PRIMVS.
ſunt vnitate,& quando non : nec alia ha-
bent tabulatæ à tranſcribendis diſcrimina,
facile eſt , è notis tabulatis tranſcribendas
colligere,vel ſola exemplorum ſequentium
imitatione. Primi ergo exempli anni DO-
MINI 1615 , ſint multipla tranſcribenda.
In primo intervallo(per primam hujus) lo-
centur 1615 quæ ſimplum ſunt; in ſecundo
ſe offerunt 2 & 1,2 & 1, 0, quæ ſunt 3230
pro dicti anni duplo; in tertio 3 & 1, 8, 3
& 1, 5, quæ ſunt 4845 pro triplo ejuſdem;
in quarto 4 & 2, 4, 4 & 2, 0, eæ ſunt
6460 pro quadruplo; in quinto 5 & 3,0, 5
& 2, 5, quæ ſunt 8075 pro quintuplo: in
ſexto 6 & 3, 6, 6 & 3, 0, quæ ſunt 9690
pro ſextuplo: in ſeptimo 7 & 4, 2, 7 & 3,
5, quæ ſunt 11305 pro ſeptuplo : in octa-
vo interſtitio 8 & 4, 8, 8 & 4, 0, quæ ſunt
12920 pro dati anni octuplo: in nono tan-
dem interſtitio ſunt 9 & 5, 4, 9 & 4, 5, quæ
ſunt 14535 pro dicti anni nocuplo. Simi-
liter ſecundi exempli tabulati ſtabit in pri-
mo ſeu ſummo tabulæ intervallo ipſum
ſimplum 166702498. Quod in ſecundo
duplum eſt, & ſic legitur & tranſcribitur
333404996. E tertio triplum ſic tranſcri
bitur 500107494. E quarto quadruplum
ſic tranſcribitur 666809992. E quinto
quintuplum dati numeri ſic tranſcribitur
833512490. E ſexto intervallo ſextuplum
ejuſdem ſic tranſcribitur 1000214988.
E ſeptimo ſeptuplum ejuſdem ſic legitur
& tranſcribitur 1166917486. Ex octavo
ſic tranſcribendum eſt octuplum oblati nu-
numeri,



CAPVT SECVNDVM. 15
meri, 1333619984. Denique propoſiti
numeri noncuplum è nono intervallo ſic
tranſcribitur 1500322482. Quę,& ſimilia
omnia brevi exercitio diſces tam antror-
ſum quam retrorſum legere, & tranſcribe-
re: nec vlla niſi in multiplorum lectione &
tranſcriptione occurrit in hac Virgulari
Arithmetica difficultas.

ADMONITIO PRO ADDITIONE
ET SVBSTRACTIONE.

*QVum difficiliorum duntaxat Arith-
meticæ operationum gratiâ inventæ
ſunt hæ Virgulæ (cujuſmodi ſunt Multiplica-
tiones, Diviſiones, Extractiones quadratæ, &
cubicæ:) Additiones autem, & Subſtractio-
nes cuivis tyrunculo ſunt obviæ: eis igitur
omiſſis, à Multiplicatione meritò ſumemus
exordium.*

CAPVT III.
De Multiplicatione.

MVLTIPLICANTIS, Multiplicandi, &
Multipli voces, ex vulgari Arithmeti-
ca patent. Quotumum autem(quaſi quo-
tulum) hîc voco, notam ſimplicem, quæ to-
ties vnitatem continet, quoties multiplum
tabulatum complectitur ſuum ſimplum.
Vnde

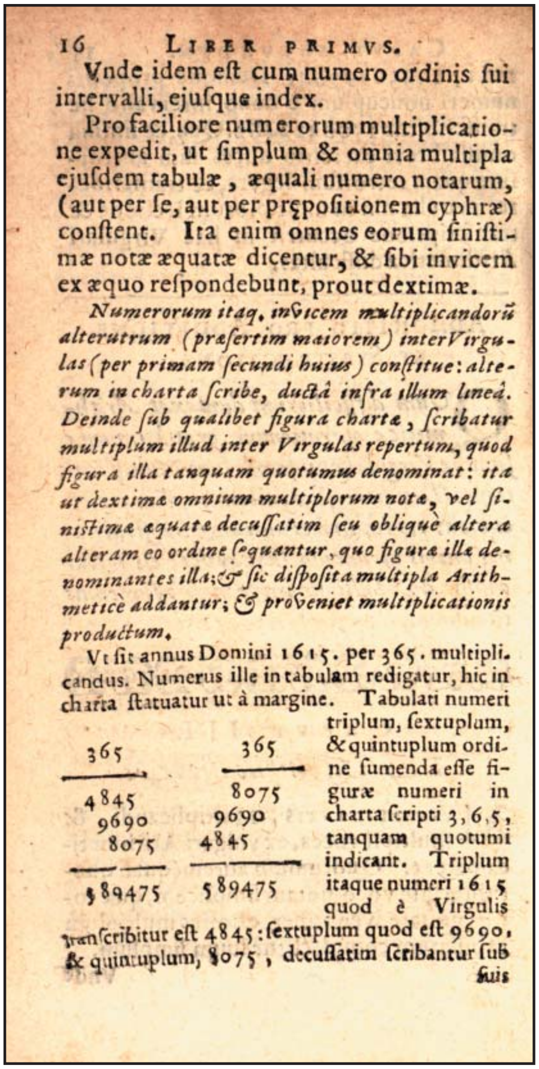
Admonitio (remark or warning) on addition and subtraction.
Napier explains that addition and subtraction are things even novices know how to accomplish (and the fact that the rods were invented to help with multiplication and division), he intends to ignore the easier operations and proceed directly to the more difficult ones.
At this point it is worth noting that multiplication was often considered a university level subject in Napier's day.

Chapter III: Multiplication.

Napier indicates that the terms *multiplier*, *multiplicand*, and *multiple* are well known, but that he will use the term *quotumus* to mean the single digit multiple that identifies any line from the rods, e.g., the fifth line on the rods will have a quotumus of 5.

From the Tomash Library on the History of Computing

Napier, John (1550–1617) *Rabdologiae*, 1617, Edinburgh



16 LIBER PRIMVS.

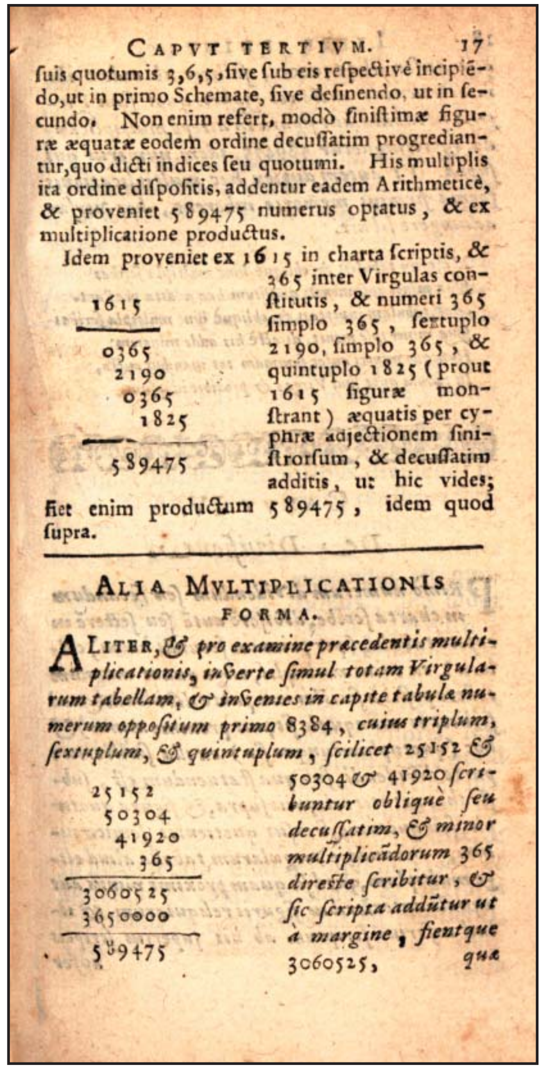
Vnde idem eſt cum numero ordinis ſui intervalli, ejuſque index.

Pro faciliore numerorum multiplicatione expedit, ut ſimplum & omnia multipla ejuſdem tabulæ, æquali numero notarum, (aut per ſe, aut per prępoſitionem cyphræ) conſtent. Ita enim omnes eorum ſiniſtimæ notæ æquatæ dicentur, & ſibi invicem ex æquo reſpondebunt, prout dextimæ.

Numerorum itaq. inuicem multiplicandorũ alterutrum (præſertim maiorem) interVirgulas (per primam ſecundi huius) conſtitue: alterum in charta ſcribe, ductâ infra illum lineâ. Deinde ſub qualibet figura chartæ, ſcribatur multiplum illud inter Virgulas repertum, quod figura illa tanquam quotumus denominat: ita ut dextimæ omnium multiplorum notæ, vel ſiniſtimæ æquatæ decuſſatim ſeu obliquè altera alteram eo ordine ſequantur, quo figuræ illæ denominantes illa; & ſic diſpoſita multipla Arithmeticè addantur; & proueniet multiplicationis productum.

Vt ſit annus Domini 1615. per 365. multiplicandus. Numerus ille in tabulam redigatur, hic in charta ſtatuatur ut à margine. Tabulati numeri triplum, ſextuplum, & quintuplum ordine ſumenda eſſe figuræ numeri in charta ſcripti 3, 6, 5, tanquam quotumi indicant. Triplum itaque numeri 1615 quod è Virgulis tranſcribitur eſt 4845: ſextuplum quod eſt 9690, & quintuplum, 8075, decuſſatim ſcribantur ſub ſuis

365	365
4845	8075
9690	9690
8075	4845
589475	589475



CAPVT TERTIVM. 17

ſuis quotumis 3,6,5, ſive ſub eis reſpectivè incipiẽdo, ut in primo Schemate, ſive deſinendo, ut in ſecundo. Non enim refert, modò ſiniſtimæ figuræ æquatæ eodem ordine decuſſatim progrediantur, quo dicti indices ſeu quotumi. His multiplis ita ordine diſpoſitis, addentur eadem Arithmeticè, & proveniet 589475 numerus optatus, & ex multiplicatione productus.

Idem proveniet ex 1615 in charta ſcriptis, & 365 inter Virgulas conſtitutis, & numeri 365 ſimplo 365, ſextuplo 2190, ſimplo 365, & quintuplo 1825 (prout 1615 figuræ monſtrant) æquatis per cyphræ adjectionem ſiniſtrorſum, & decuſſatim additis, ut hic vides; fiet enim productum 589475, idem quod ſupra.

1615
0365
2190
0365
1825
589475

ALIA MVLTIPLICATIONIS FORMA.

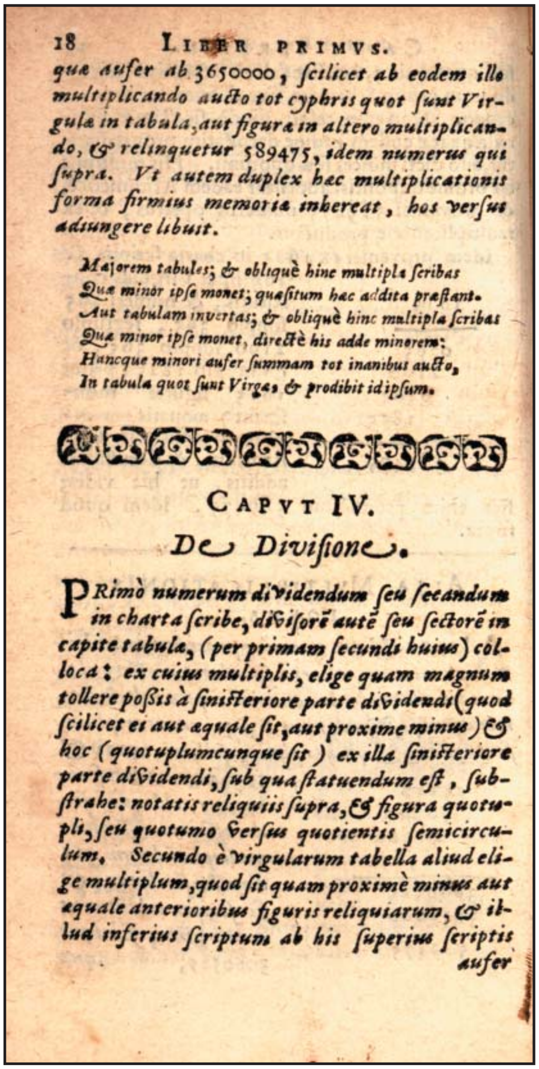
A*LITER, & pro examine præcedentis multiplicationis, inverte ſimul totam Virgularum tabellam, & invenies in capite tabulæ numerum oppoſitum primo* 8384, *cuius triplum, ſextuplum, & quintuplum, ſcilicet* 25152 *&* 50304 *&* 41920 *ſcribuntur obliquè ſeu decuſſatim, & minor multiplicãdorum* 365 *directè ſcribitur, & ſic ſcripta addũtur ut à margine, fientque* 3060525, *quæ*

25152
50304
41920
365
3060525
3650000
589475

Napier shows how the rods may be used to determine any single digit multiple of the multiplicand and then these may be added up (appropriately shifted by a decimal place) to give the final product. He gives an example of 1615 * 365. He does not write the multiplicand, but suggests writing the multiplier (365) and under it writing the 3, 6 and 5 multiples (found from the rods of 1615) and then adding these up to give the product 589475. He then shows that the product 365 * 1615 yields the same result.

As a check on the results, you can turn the entire block of rods over (thus exposing the complement numbers on the opposite faces) to find that they now represent the number 8384 rather than the original number of 1615. Add 1 making it 8385. Multiplying this new number by 365 will yield the product 3,060,525. Subtract this product from 365, to which has been appended as many zeros as there were rods on the table (4) and the result will be 589475 - the original product of 1615 * 365.

Napier, John (1550–1617) *Rabdologiae*, 1617, Edinburgh



18 LIBER PRIMVS.

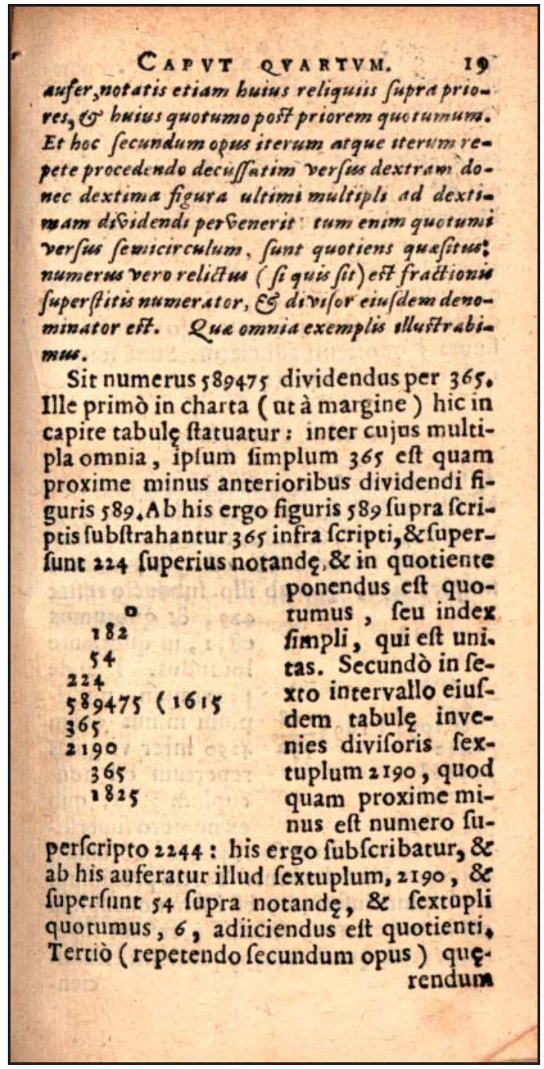
quæ auſer ab 3650000, ſcilicet ab eodem illo multiplicando aucto tot cyphris quot ſunt Virgulæ in tabula, aut figuræ in altero multiplicando, & relinquetur 589475, idem numerus qui ſupra. Vt autem duplex hæc multiplicationis forma firmius memoriæ inhereat, hos verſus adiungere libuit.

Majorem tabules; & obliquè hinc multipla ſcribas
Quæ minor ipſe monet; quæſitum hæc addita præſtant.
Aut tabulam invertas; & obliquè hinc multipla ſcribas
Quæ minor ipſe monet, directè his adde minorem:
Hancque minori auſer ſummam tot inanibus aucto,
In tabula quot ſunt Virgæ & prodibit idipſum.

CAPVT IV.

De Diviſione.

PRimò numerum dividendum ſeu ſecandum in charta ſcribe, diviſorē autē ſeu ſectorē in capite tabulæ, (per primam ſecundi huius) colloca: ex cuius multiplis, elige quam magnum tollere poſſis à ſiniſteriore parte dividendi (quod ſcilicet ei aut æquale ſit, aut proxime minus) & hoc (quotuplumcunque ſit) ex illa ſiniſteriore parte dividendi, ſub qua ſtatuendum eſt, ſubſtrahe: notatis reliquiis ſupra, & figura quotupli, ſeu quotumo verſus quotientis ſemicirculum. Secundo è virgularum tabella aliud elige multiplum, quod ſit quam proximè minus aut æquale anterioribus figuris reliquiarum, & illud inferius ſcriptum ab his ſuperius ſcriptis auſer



CAPVT QVARTVM. 19

auſer, notatis etiam huius reliquiis ſupra priores, & huius quotumo poſt priorem quotumum. Et hoc ſecundum opus iterum atque iterum repete procedendo decuſſatim verſus dextram donec dextima figura ultimi multipli ad dextimam dividendi pervenerit: tum enim quotumi verſus ſemicirculum, ſunt quotiens quæſitus: numerus vero relictus (ſi quis ſit) eſt fractionis ſuperſtitis numerator, & diviſor eiuſdem denominator eſt. Quæ omnia exemplis illuſtrabimus.

Sit numerus 589475 dividendus per 365. Ille primò in charta (ut à margine) hic in capite tabulę ſtatuatur: inter cujus multipla omnia, ipſum ſimplum 365 eſt quam proxime minus anterioribus dividendi figuris 589. Ab his ergo figuris 589 ſupra ſcriptis ſubſtrahantur 365 infra ſcripti, & ſuperſunt 224 ſuperius notandę, & in quotiente ponendus eſt quotumus, ſeu index ſimpli, qui eſt unitas. Secundò in ſexto intervallo eiuſdem tabulę invenies diviſoris ſextuplum 2190, quod quam proxime minus eſt numero ſuperſcripto 2244: his ergo ſubſcribatur, & ab his auferatur illud ſextuplum, 2190, & ſuperſunt 54 ſupra notandę, & ſextupli quotumus, 6, adiiciendus eſt quotienti. Tertiò (repetendo ſecundum opus) quę-rendum

```
      0
    182
    54
  224
  589475 (1615
  365
  2190
    365
    1825
```

The italicized material immediately above the start of Chapter IV is a set of easily memorized rules for the previous multiplication operations.

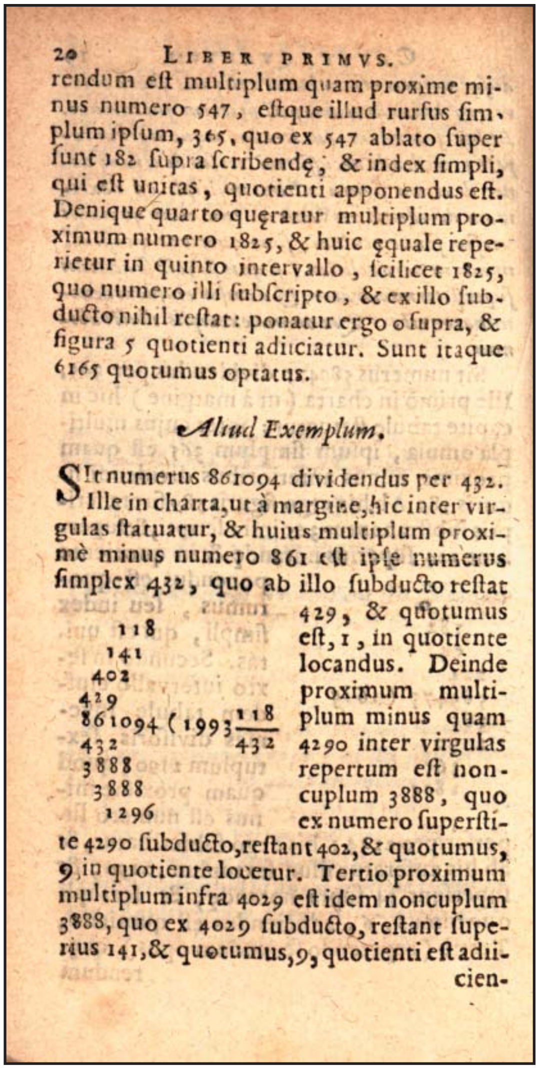
Chapter IV: Division.

To divide one number (the dividend 58975) by another (the divisor 365) write down the dividend and set up the rods to represent the divisor. Examine the rows of the rods to determine the largest multiple of the divisor that is less than or equal to the first digits of the dividend (the first multiple, 365, is the one less than the first 3 digits, 589, of the dividend). Write that multiple under the dividend and subtract it from those first digits, writing the remainder (224, because 589 - 365 = 224) above the dividend and write the *quotumus* (the row from which the multiple was taken - in this case 1) to the right of the dividend.

Proceed as above, only now attempt to find the multiple of 365 that is less than or equal to 2244 (the 224 being the remainder written above the dividend and the final 4 being taken from the next digit of the actual dividend).

As can be seen, this is essentially the same method of dividing that we use with paper and pencil today (long division), except the layout of the results is the form used in Napier’s day. Readers in the 1600s would have been used to a form of division known as *galley division* (because the resulting diagram of digits was thought to resemble a galley under sail - and examples were concocted where the illustration of the galley was remarkably good) and thus would have had no problem understanding Napier’s notation and the placement of the various numbers.

Napier, John (1550–1617) *Rabdologiae*, 1617, Edinburgh



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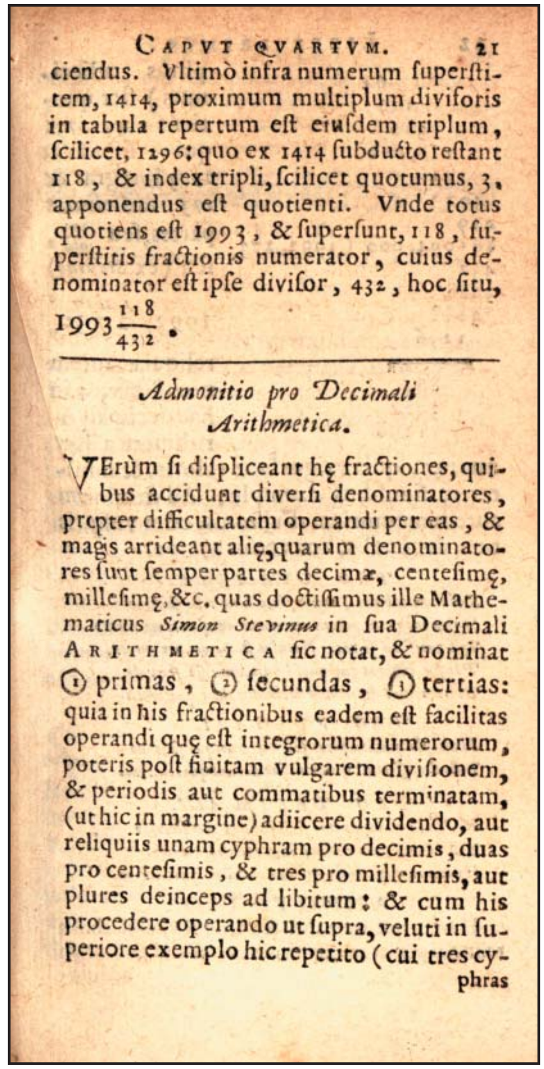
rendum est multiplum quam proxime minus numero 547, estque illud rursus simplum ipsum, 365, quo ex 547 ablato super sunt 182 supra scribendę, & index simpli, qui est unitas, quotienti apponendus est. Denique quarto quęratur multiplum proximum numero 1825, & huic ęquale reperietur in quinto intervallo, scilicet 1825, quo numero illi subscripto, & ex illo subducto nihil restat: ponatur ergo o supra, & figura 5 quotienti adiiciatur. Sunt itaque 6165 quotumus optatus.

Aliud Exemplum.

SIt numerus 861094 dividendus per 432. Ille in charta, ut à margine, hic inter virgulas statuatur, & huius multiplum proximè minus numero 861 est ipse numerus simplex 432, quo ab illo subducto restat 429, & quotumus est, 1, in quotiente locandus. Deinde proximum multiplum minus quam 4290 inter virgulas repertum est noncuplum 3888, quo ex numero superstite 4290 subducto, restant 402, & quotumus, 9, in quotiente locetur. Tertio proximum multiplum infra 4029 est idem noncuplum 3888, quo ex 4029 subducto, restant superius 141, & quotumus, 9, quotienti est adiicien-

118
141
402
429
861094 ($1993\frac{118}{432}$
432
3888
3888
1296



CAPVT QVARTVM. 21

ciendus. Vltimò infra numerum superstitem, 1414, proximum multiplum divisoris in tabula repertum est eiusdem triplum, scilicet, 1296: quo ex 1414 subducto restant 118, & index tripli, scilicet quotumus, 3, apponendus est quotienti. Vnde totus quotiens est 1993, & supersunt, 118, superstitis fractionis numerator, cuius denominator est ipse divisor, 432, hoc situ, $1993\frac{118}{432}$.

Admonitio pro Decimali Arithmetica.

VErùm si displiceant hę fractiones, quibus accidunt diversi denominatores, propter difficultatem operandi per eas, & magis arrideant alię, quarum denominatores sunt semper partes decimæ, centesimę, millesimę, &c. quas doctissimus ille Mathematicus *Simon Stevinus* in sua Decimali ARITHMETICA sic notat, & nominat ① primas, ② secundas, ③ tertias: quia in his fractionibus eadem est facilitas operandi quę est integrorum numerorum, poteris post finitam vulgarem divisionem, & periodis aut commatibus terminatam, (ut hic in margine) adiicere dividendo, aut reliquiis unam cyphram pro decimis, duas pro centesimis, & tres pro millesimis, aut plures deinceps ad libitum: & cum his procedere operando ut supra, veluti in superiore exemplo hic repetito (cui tres cyphras

Another example, this time one involving a fraction in the resulting quotient: 861094 / 432 = $1993\ ^{118}/_{432}$

Admonitio pro Decimals Arithmetica

Napier mentions Simon Stevin, a dutch military man, who had published a book extolling the virtues of decimal fraction notation. In these very early days of decimal fractions the concept of using a single decimal point to delineate the fractional portion of a number was yet little known - Napier used the decimal point in one of his publications, but few others even recognized it. Those who were aware of decimal fractions used a number of different notations, for example, to write 3.1415, they might have used:

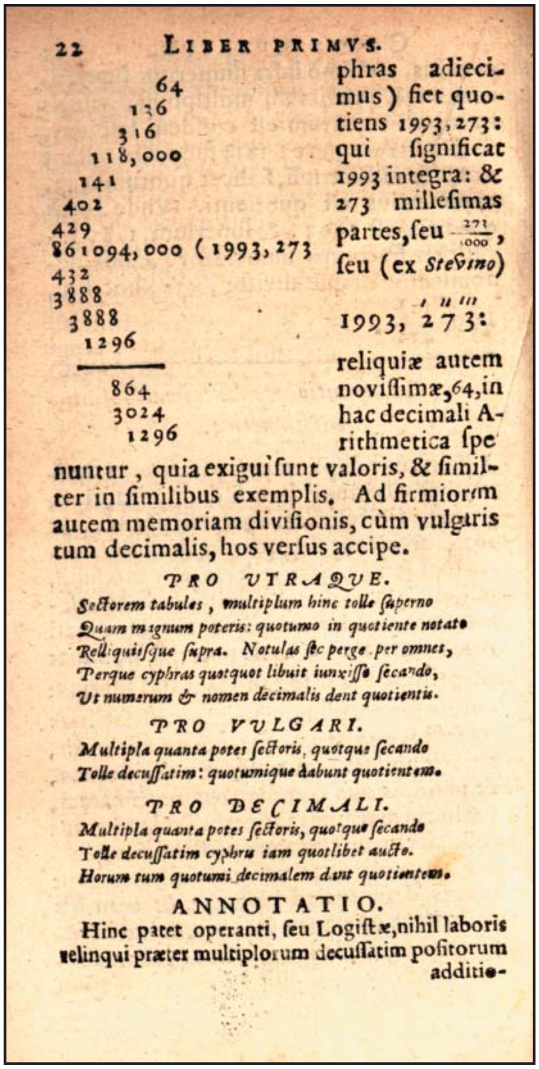
3⓪1①4②1③5④

or

3 1'4"1'"5""

or even combined these with a comma or decimal point to use:

3,1'4"1'"5""



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64	phras adieci-
136	mus) fiet quo-
316	tiens 1993,273:
118,000	qui ſignificat
141	1993 integra: &
402	273 milleſimas
429	partes,ſeu $\frac{271}{1000}$,
861094,000 (1993,273	ſeu (ex *Stevino*)
432	
3888	′ ″ ‴
3888	1993, 2 7 3:
1296	
———	reliquiæ autem
864	noviſſimæ,64,in
3024	hac decimali A-
1296	rithmetica ſpe-

nuntur, quia exigui ſunt valoris, & ſimiliter in ſimilibus exemplis. Ad firmiorem autem memoriam diviſionis, cùm vulgaris tum decimalis, hos verſus accipe.

PRO VTRAQVE.

Sectorem tabules, multiplum hinc tolle ſuperno
Quam magnum poteris: quotumo in quotiente notato
Relliquisque ſupra. Notulas ſic perge per omnes,
Perque cyphras quotquot libuit iunxiſſe ſecando,
Ut numerum & nomen decimalis dent quotientis.

PRO VVLGARI.

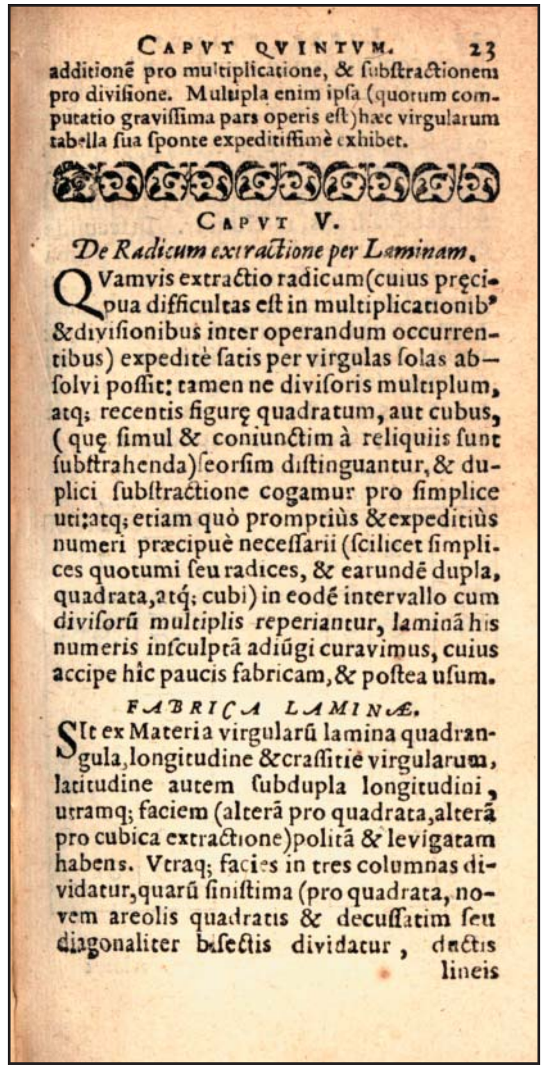
Multipla quanta potes ſectoris, quotque ſecando
Tolle decuſſatim: quotumique dabunt quotientem.

PRO DECIMALI.

Multipla quanta potes ſectoris, quotque ſecando
Tolle decuſſatim cyphris iam quotlibet aucto.
Horum tum quotumi decimalem dant quotientem.

ANNOTATIO.

Hinc patet operanti, ſeu Logiſtæ, nihil laboris relinqui præter multiplorum decuſſatim poſitorum additio-



CAPVT QVINTVM. 23

additionē pro multiplicatione, & ſubſtractionem pro diviſione. Multipla enim ipſa (quorum computatio graviſſima pars operis eſt) hæc virgularum tabella ſua ſponte expeditiſſimè exhibet.

CAPVT V.

De Radicum extractione per Laminam.

QVamvis extractio radicum (cuius præcipua difficultas eſt in multiplicationib⁹ & diviſionibus inter operandum occurrentibus) expeditè ſatis per virgulas ſolas abſolvi poſſit: tamen ne diviſoris multiplum, atq; recentis figuræ quadratum, aut cubus, (quæ ſimul & coniunctim à reliquiis ſunt ſubſtrahenda) ſeorſim diſtinguantur, & duplici ſubſtractione cogamur pro ſimplice uti; atq; etiam quò promptiùs & expeditiùs numeri præcipuè neceſſarii (ſcilicet ſimplices quotumi ſeu radices, & earundē dupla, quadrata, atq; cubi) in eodē intervallo cum diviſorū multiplis reperiantur, laminā his numeris inſculptā adiūgi curavimus, cuius accipe hîc paucis fabricam, & poſtea uſum.

FABRICA LAMINÆ.

SIt ex Materia virgularū lamina quadrangula, longitudine & craſſitie virgularum, latitudine autem ſubdupla longitudini, utramq; faciem (alterā pro quadrata, alterā pro cubica extractione) politā & levigatam habens. Vtraq; facies in tres columnas dividatur, quarū ſiniſtima (pro quadrata, novem areolis quadratis & decuſſatim ſeu diagonaliter biſectis dividatur, ductis lineis

Napier illustrates decimal notation with the earlier problem in which he obtained a fractional quotient. He gives both the fractional form of the quotient 1993 $^{271}/_{1000}$ as well as his own decimal form of 1993,273 and (as he says ‘following Stevin’) 1993,2'7"3'"

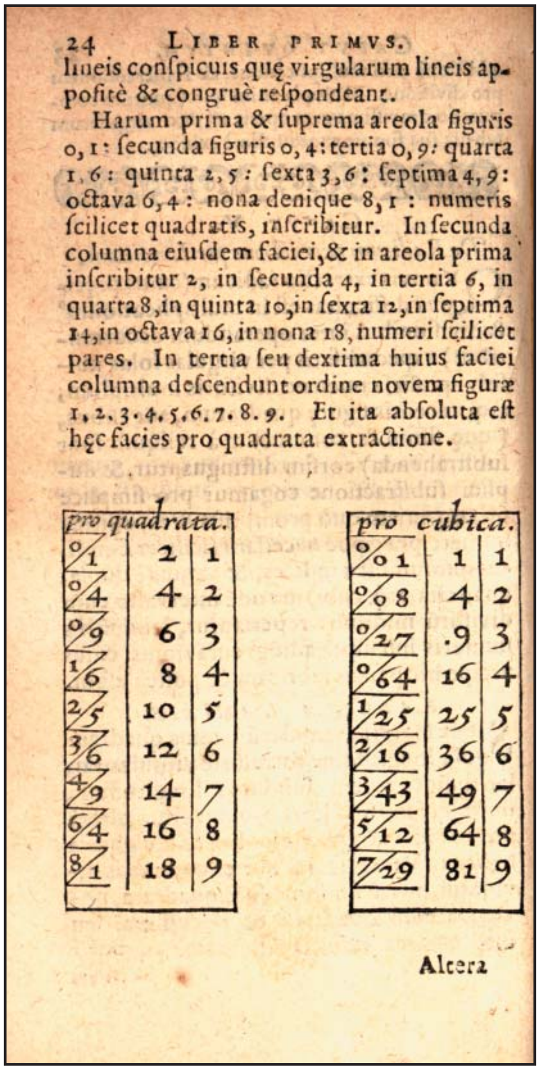
He ends this chapter with three short verses (something he and others of his time considered easy to memorize) that contained the rules for division by using the rods.

Chapter V: Rods for extracting square roots.

Napier indicates that the extraction of square and cube roots could be done, via the usual methods, with only using the rods. However it requires keeping track of several items and he has simplified the process with the creation of two special rods.

Making the rods

Create two rods (one for square and one for cube roots) as before, but make them three times as wide as the ones described earlier. Divide each rod into three columns as shown in the diagram on the next page.



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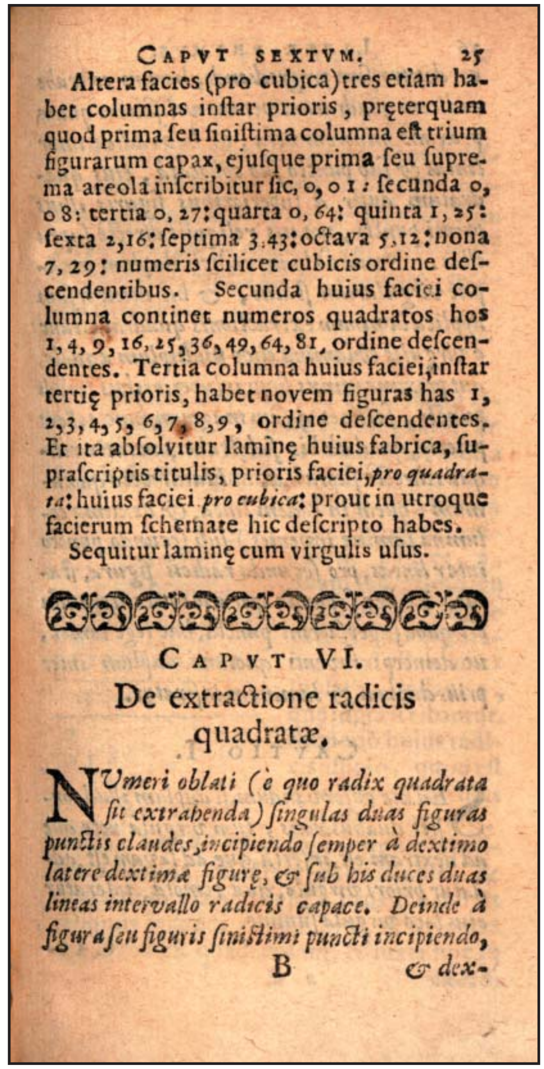
lineis conſpicuis quę virgularum lineis appoſitè & congruè reſpondeant.

Harum prima & ſuprema areola figuris o, 1: ſecunda figuris o, 4: tertia o, 9: quarta 1, 6: quinta 2, 5: ſexta 3, 6: ſeptima 4, 9: octava 6, 4: nona denique 8, 1: numeris ſcilicet quadratis, inſcribitur. In ſecunda columna eiuſdem faciei, & in areola prima inſcribitur 2, in ſecunda 4, in tertia 6, in quarta 8, in quinta 10, in ſexta 12, in ſeptima 14, in octava 16, in nona 18, numeri ſcilicet pares. In tertia ſeu dextima huius faciei columna deſcendunt ordine novem figuræ 1, 2, 3, 4, 5, 6, 7, 8, 9. Et ita abſoluta eſt hęc facies pro quadrata extractione.

pro quadrata.		
0/1	2	1
0/4	4	2
0/9	6	3
1/6	8	4
2/5	10	5
3/6	12	6
4/9	14	7
6/4	16	8
8/1	18	9

pro cubica.		
0/01	1	1
0/08	4	2
0/27	9	3
0/64	16	4
1/25	25	5
2/16	36	6
3/43	49	7
5/12	64	8
7/29	81	9

Altera



CAPVT SEXTVM. 25

Altera facies (pro cubica) tres etiam habet columnas inſtar prioris, pręterquam quod prima ſeu ſiniſtima columna eſt trium figurarum capax, ejuſque prima ſeu ſuprema areola inſcribitur ſic, o, o 1: ſecunda o, o 8: tertia o, 27: quarta o, 64: quinta 1, 25: ſexta 2, 16: ſeptima 3, 43: octava 5, 12: nona 7, 29: numeris ſcilicet cubicis ordine deſcendentibus. Secunda huius faciei columna continet numeros quadratos hos 1, 4, 9, 16, 25, 36, 49, 64, 81, ordine deſcendentes. Tertia columna huius faciei, inſtar tertię prioris, habet novem figuras has 1, 2, 3, 4, 5, 6, 7, 8, 9, ordine deſcendentes. Et ita abſolvitur laminę huius fabrica, ſupraſcriptis titulis, prioris faciei, *pro quadrata*: huius faciei *pro cubica*: prout in utroque facierum ſchemate hic deſcripto habes.

Sequitur laminę cum virgulis uſus.

CAPVT VI.

De extractione radicis quadratæ.

Numeri oblati (è quo radix quadrata ſit extrahenda) ſingulas duas figuras punctis claudes, incipiendo ſemper à dextimo latere dextimæ figurę, & ſub his duces duas lineas intervallo radicis capace. Deinde à figura ſeu figuris ſiniſtimi puncti incipiendo,

B & dex-

For the square root rod (labelled *pro quadrata*) the left hand column contains (in the same way as the earlier rods) the squares of the integers. The right hand column contains the digits from 1 to 9 while the middle column contains the values that are twice those in the right hand column.

The cube root rod (*pro cubica*) is inscribed the same way, only the left hand column contains the cubes of the integers and the central column contains their squares.

Chapter VI: Extracting a square root.

Write down the number whose root is to be found and, starting from the right, put a dot between each pair of digits (see the example on page 27).

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Napier, John (1550–1617) *Rabdologiae*, 1617, Edinburgh

& dextrorſum progrediendo, harum extrahe radicem quadratam veram vel ſaltem quam proximè minorem vera, & hac radice inter lineas & ſub puncto ſuo collocatâ, eius quadratum aufer à ſuperioribus figuris illius primi puncti, notatis reliquiis directè ſupra illas. Secundò huius radicis duplum in capite virgularum ſtatue, & his dextrorſum applica laminam extractionis quadratæ: tunc è virgulis & lamina elice multiplum æquale, aut proximè minus figuris ſuperioribus ſecũdi puncti, ſcilicet quàm magnum hinc tollere poteris, quod ab his ſubſtrahe, notatis reliquiis directè ſupra has. Huius vero quotumum (quem in eadem linea, & dextima columna laminæ invenies) ſub ſecundo puncto inter lineas, pro ſecunda radicis figura, ſtatue. Et hęc ſecunda operatio toties iteranda eſt quot ſuperfuerint puncta, hac lege tamen, ut deinceps inventi quotumi duplum inter prius duplum, & laminam inſeratur.

CAVTIO I.

SEd hic obſervandum, ſi duplum illud conſtet duabus notis, tum virgulâ notę quę ad dextram eſt, inſertâ, quę ad lævam eſt, addatur priori virgulæ, quâ remotâ, inſeratur ejus loco virgula ſummę.

CAV-

CAVTIO II.

RVrſus hic obſervandum eſt, quod ſi nullum ex multiplis, imo ne ſimplum quidem auferendum ex figuris præſens punctum præeuntibus, ab eis auferri poſſit: ponenda eſt o cyphra ſub puncto illo pro quotumo, intactis reliquiis.

EXEMPLVM.

SIt numeri 117716237694 extrahenda radix quadrata. Punctis diſtinguatur, & ſub eo ducantur lineę, vt à margine: inde à figuris ſiniſtimi puncti, videlicet 11, elice radicem quadratam quàm proximè minorem ſcilicet, 3, quam ſub primo ſeu ſiniſtimo puncto ſtatue, & eius quadrato, quod eſt 9, ab 11 ſublato, reſtant 2, quę ſupra ſcribantur. Secundò huius radicis duplo, quod eſt 6, in capite alicuius virgulę invento, huic virgulę applicetur laminæ quadratę extractionis, & quęratur in eis multiplum proximè minus reliquiis ſecundi puncti, 277, & invenies 256, quod eſt quadruplum, & iuxta hoc in

```
                 90
          54895
      67
   21
 2
11.77.16.23.76.94.
__________________
 3   4   3   0   9   8
__________________
9
256
  2049
      617481
        5489504
```

 eadem

Draw two parallel lines under the dotted number, leaving enough room to write down the figures which will be the square root.

/6	0/1	2	1
1/2	0/4	4	2
1/8	0/9	6	3
2/4	1/6	8	4
3/0	2/5	10	5
3/6	3/6	12	6
4/2	4/9	14	7
4/8	6/4	16	8
5/4	8/1	18	9

Starting at the left, find the square root whose square is less than or equal to the first pair of digits (or single digit if there is only one). In this example the first pair of digits is 11 and the required square root is 3 (9 being the square just less than 11).

Write the square (9) under the parallel lines and write the root digit (3) under the first dot then subtract the square (9) from the pair being considered (11) and write the remainder (2) above the pair. Napier does not provide a diagram for the next steps, so one is provided here.

Double the root digit just obtained (3 doubled is 6 which is conveniently adjacent in the middle column of the square root bone) and place the usual rods for this number to the right of the square root rod. Find the number composed of the digits of the remainder you wrote down (2) and the next two dotted digits (77) (277 in this example) and determine the row (using the added bones and the square root bone) that contains a number equal to or less than that value. The number is found in the fourth row (256). Write down the row in which it was found (4) between the lines and the square (256) under the lines, subtract that square (256) from the number above (277) and write the remainder (21) above the lines as illustrated.

Repeat the above process for each pair of dotted digits, each time adding the rods that represent twice the row number (8 for this last step) between the previous double and the square root plate. Thus for the third step you will have the rods of 6, 8, and the square root rod in that order. At the fourth step you will find that no number on the assembled rods can be found that is less than or equal to the one sought (6,723) so you must enter a 0 as the next digit between the lines and a rod for 0 must be inserted beside the square root rod. The fifth step is to search for something less or equal to 672,376 and you will find that on the 9th line. Double 9 is 18 so

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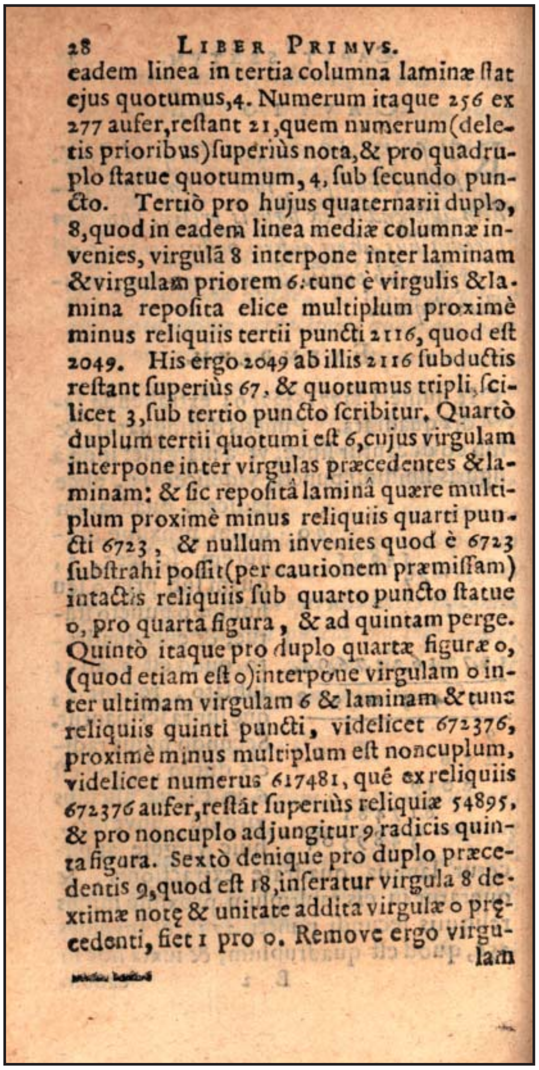

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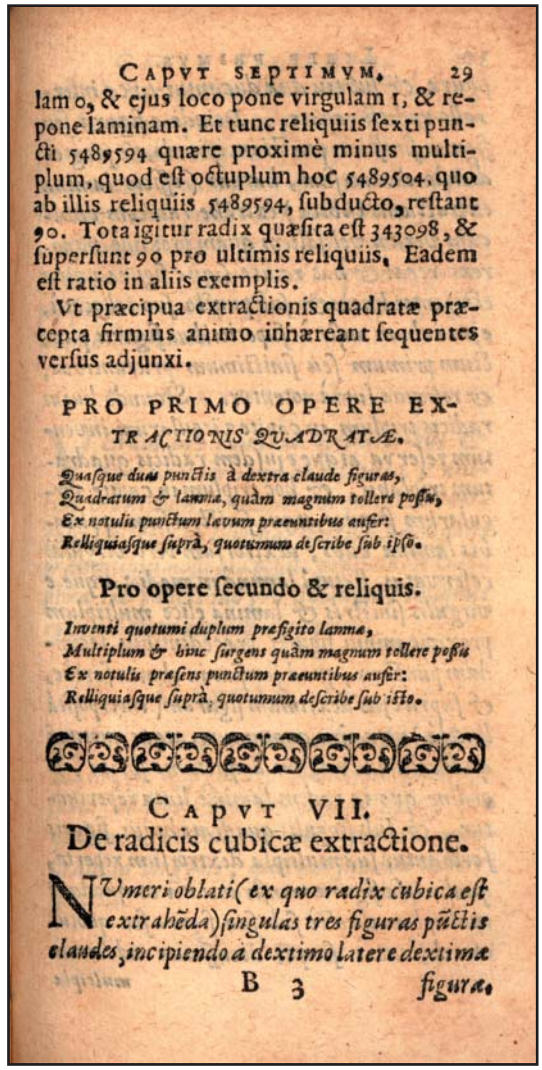
eadem linea in tertia columna laminæ ſtat ejus quotumus,4. Numerum itaque 256 ex 277 aufer,reſtant 21,quem numerum(deletis prioribus)ſuperiùs nota,& pro quadruplo ſtatue quotumum, 4, ſub ſecundo puncto. Tertiò pro hujus quaternarii duplo, 8,quod in eadem linea mediæ columnæ invenies, virgulã 8 interpone inter laminam &virgulam priorem 6: tunc è virgulis &lamina repoſita elice multiplum proximè minus reliquiis tertii puncti 2116, quod eſt 2049. His ergo 2049 ab illis 2116 ſubductis reſtant ſuperiùs 67, & quotumus tripli, ſcilicet 3,ſub tertio puncto ſcribitur. Quartò duplum tertii quotumi eſt 6,cujus virgulam interpone inter virgulas præcedentes &laminam: & ſic repoſitâ laminâ quære multiplum proximè minus reliquiis quarti puncti 6723 , & nullum invenies quod è 6723 ſubſtrahi poſſit(per cautionem præmiſſam) intactis reliquiis ſub quarto puncto ſtatue o, pro quarta figura , & ad quintam perge. Quintò itaque pro duplo quartæ figuræ o, (quod etiam eſt o)interpone virgulam o inter ultimam virgulam 6 & laminam & tunc reliquiis quinti puncti, videlicet 672376, proximè minus multiplum eſt noncuplum, videlicet numerus 617481, quē ex reliquiis 672376 aufer,reſtãt ſuperiùs reliquiæ 54895, & pro noncuplo adjungitur 9 radicis quinta figura. Sextò denique pro duplo præcedentis 9,quod eſt 18,inſeratur virgula 8 dextimæ notę & unitate addita virgulæ o præcedenti, fiet 1 pro o. Remove ergo virgulam



CAPVT SEPTIMVM. 29

lam o, & ejus loco pone virgulam 1, & repone laminam. Et tunc reliquiis ſexti puncti 5489594 quære proximè minus multiplum, quod eſt octuplum hoc 5489504. quo ab illis reliquiis 5489594, ſubducto, reſtant 90. Tota igitur radix quæſita eſt 343098 , & ſuperſunt 90 pro ultimis reliquiis. Eadem eſt ratio in aliis exemplis.

Vt præcipua extractionis quadratæ præcepta firmiùs animo inhæreant ſequentes verſus adjunxi.

PRO PRIMO OPERE EXTRACTIONIS QVADRATÆ.

Quaſque duas punctis à dextra claude figuras,
Quadratum & lamnæ, quàm magnum tollere poſſis,
Ex notulis punctum lævum praeuntibus aufer:
Relliquiaſque ſuprà, quotumum deſcribe ſub ipſo.

Pro opere ſecundo & reliquis.

Inventi quotumi duplum præfigito lamnæ,
Multiplum & hinc ſurgens quàm magnum tollere poſſis
Ex notulis præſens punctum praeuntibus aufer:
Relliquiaſque ſuprà, quotumum deſcribe ſub isto.

CAPVT VII.

De radicis cubicæ extractione.

NVmeri oblati (ex quo radix cubica eſt extrahēda) ſingulas tres figuras pūctis claudes, incipiendo à dextimo latere dextimæ

B 3 figuræ.

insert the 8 rod beside the square root rod and "carry the 1" to the next rod to the left (i.e., make the rod to the left a 1 rod rather than it remaining a 0 rod) At the end you will have the rods for 6, 8, 6, 1, 8, and the square root rod in that order.

Napier again appends some Latin verse to the end of these instructions as an aid to memorizing the steps.

Chapter VII Extracting the cube root.

This process is very similar to that of extracting the square root. One, of course, uses the cube root rod and marks off the digits with dots in groups of three beginning with the right hand end of the number. The rest of the process, while following the pattern set in the square root extraction, is burdened with putting rods to both the right and left of the cube root bone, making trial computations and rejecting some while saving others at each step. These trial calculations are shown on page 33 for the example that follows.

The whole process is difficult to explain without a series of examples done on the rods and, as Fermat once remarked "the margin is not big enough to contain the work." If it is imperative to find the method, then we suggest consulting the English translation of this work:

Napier, John, *Rabdology*, Translated by William Frank Richardson, Charles Babbage Institute Reprint Series for the History Of Computing, Vol. 15. MIT Press (Cambridge, Mass., 1990).

It should be pointed out that Napier's own description is difficult to follow and, like the original, this translation contains no diagrams to aid the reader.

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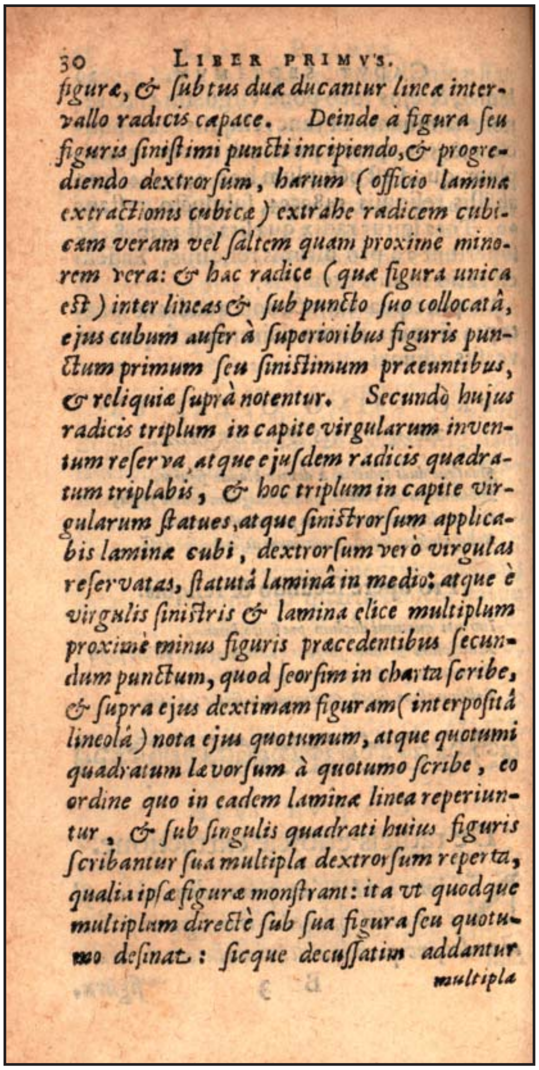

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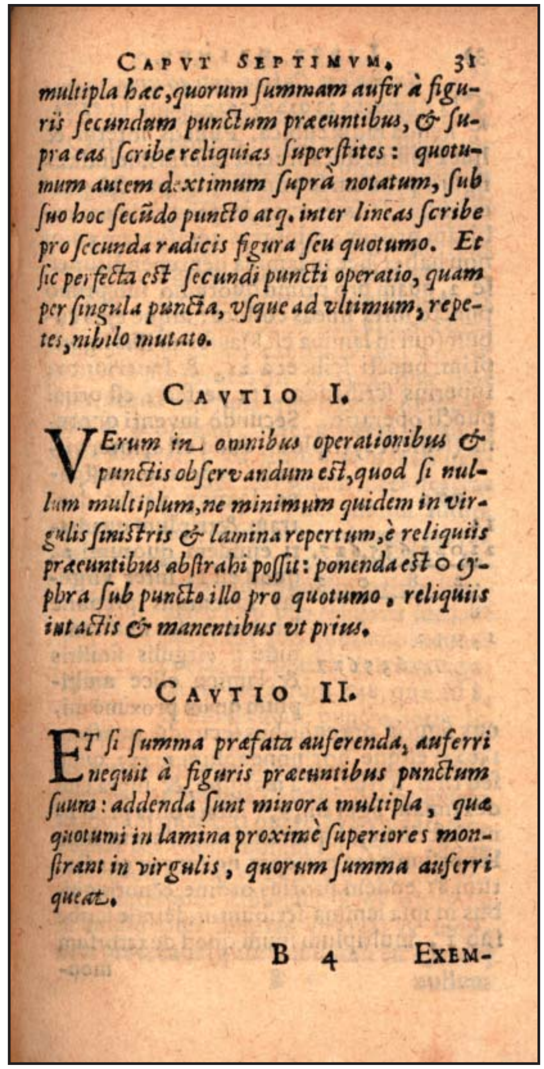
figuræ, & ſubtus duæ ducantur lineæ intervallo radicis capace. Deinde à figura ſeu figuris ſiniſtimi puncti incipiendo, & progrediendo dextrorſum, harum (officio laminæ extractionis cubicæ) extrahe radicem cubicam veram vel ſaltem quam proximè minorem vera: & hac radice (quæ figura unica eſt) inter lineas & ſub puncto ſuo collocatâ, ejus cubum aufer à ſuperioribus figuris punctum primum ſeu ſiniſtimum præeuntibus, & reliquiæ ſuprà notentur. Secundò hujus radicis triplum in capite virgularum inventum reſerva, atque ejuſdem radicis quadratum triplabis, & hoc triplum in capite virgularum ſtatues, atque ſiniſtrorſum applicabis laminæ cubi, dextrorſum verò virgulas reſervatas, ſtatutâ laminâ in medio: atque è virgulis ſiniſtris & lamina elice multiplum proximè minus figuris præcedentibus ſecundum punctum, quod ſeorſim in charta ſcribe, & ſupra ejus dextimam figuram (interpoſitâ lineolâ) nota ejus quotumum, atque quotumi quadratum lævorſum à quotumo ſcribe, eo ordine quo in eadem laminæ linea reperiuntur, & ſub ſingulis quadrati huius figuris ſcribantur ſua multipla dextrorſum reperta, qualia ipſæ figuræ monſtrant: ita vt quodque multiplum directè ſub ſua figura ſeu quotumo deſinat: ſicque decuſſatim addantur

multipla



CAPVT SEPTIMVM. 31

multipla hæc, quorum ſummam aufer à figuris ſecundum punctum præeuntibus, & ſupra eas ſcribe reliquias ſuperſtites: quotumum autem dextimum ſuprà notatum, ſub ſuo hoc ſecũdo puncto atq. inter lineas ſcribe pro ſecunda radicis figura ſeu quotumo. Et ſic perfecta eſt ſecundi puncti operatio, quam per ſingula puncta, vſque ad vltimum, repetes, nihilo mutato.

CAVTIO I.

VErum in omnibus operationibus & punctis obſervandum eſt, quod ſi nullum multiplum, ne minimum quidem in virgulis ſiniſtris & lamina repertum, è reliquiis præeuntibus abſtrahi poſſit: ponenda eſt 0 cyphra ſub puncto illo pro quotumo, reliquiis intactis & manentibus vt prius.

CAVTIO II.

ET ſi ſumma præfata auferenda, auferri nequit à figuris præeuntibus punctum ſuum: addenda ſunt minora multipla, quæ quotumi in lamina proximè ſuperiores monſtrant in virgulis, quorum ſumma auferri queat.

B 4 EXEM-

The *Cautio* (caution) I and II are simply an addenda to the general rules for finding cube roots for the instances when no multiples can be found on the bones for a particular step.

Exemplum Cubicæ extractionis.

SIT numerus 22022635627, à quo ſit extrahenda radix cubica. Punctis notetur, & lineæ ſubtus ducantur, ut inferius: deinde ex figuris primum ſeu ſiniſtimum punctũ præeuntibus, ſcilicet ex 22, extrahe radicem cubicam proximè minorem vera (veram enim non habet) hęc in lamina deprehenditur eſſe 2, quam pro primo quotumo ſub primo puncto inter lineas colloca: atque ejus cubum (qui in lamina eſt 8) aufer ab illis figuris primi puncti ſcilicet à 22, & ſuperſunt 14 ſuperiùs ſcribenda. Ita perfecta eſt primi puncti operatio. Secundò inventi quotumi (ſcilicet 2) triplum, quod eſt 6, inter virgulas repertum poſtpone laminę verſus dextram, & triplum quadrati ejuſdem quotumi 2, quod eſt 12 inter virgulas inventum prępone laminæ verſus ſiniſtram: inde è virgulis ſiniſtris & lamina elice multiplum quàm proximè minus figuris præeuntibus ſecundũ punctum 14022; eſtque hoc noncuplum 11529, quod ſeorſim ſcribe, ut à margine, & ſupra ejus dextimam figuram, 9, (interpoſita prius linea) ſcribe ejus quotumum 9; atque hinc lævorſum nota ejuſdem novenarii quadratum 81, eodem prorſus ordine, & notis quibus in ipſa lamina ſcribuntur; deinde ſcribe ſub 1, multiplum ſuum quod dextrorſum mon-

```
   070
14
22.022.635.627.
 ______________
  2   8   0   3
 ______________
 8
13952
      70635627
```


monſtrat, quod eſt ſimplum 6, & ſub 8 ſcribe multiplum quod dextrorſum monſtrat quod eſt octuplum 48: & hæc tria multipla ſic decuſſatim infra lineam ſcripta & addita (ut à margine) producunt 16389, quę quia à ſuperioribus figuris 14022 ſecundi puncti auferri nequiunt, repudiandus eſt novenarius & loco 819 (per cautionem ſecundam) capiendæ ſunt notulæ proximè ſuperiores in lamina, quæ ſunt 648: atque multipla quæ hæ demonſtrant, ſcilicet octuplum, inter ſiniſtras virgulas quod eſt 10112, & quadruplum inter dextras quod eſt 24, & ſextuplum inter dextras quod eſt 36, decuſſatim addita (ut à margine) producunt 13952: quibus ex 14022 ſubductis, remanet ſuperiùs (in primo ſchemate) 70 pro reliquiis ſecundi puncti, & pro quotumo ſecundi puncti accipiatur dextima figurarum electarum 648, quæ eſt 8, & ſub ſecundo puncto inter lineas ſtatuatur. Tertiò quotumorum pręcedentium (ſcilicet 28) triplum, quod eſt 84, pone per virgulas à dextris: & eorundem 28 triplum quadrati quęre, ſive vulgari modo, ſive per compendium ſequens, eſtque 2352: quod officio virgularum à ſiniſtris pone, & interpone laminam. Et ex multiplis & ſimplo inter ſiniſtras virgulas & laminam procreatis (quorũ minimũ eſt 235201) nullum

```
  819
 _____
11529
    6
   48
 _____
16389

  648
 _____
10112
   24
   36
 _____
13952
```

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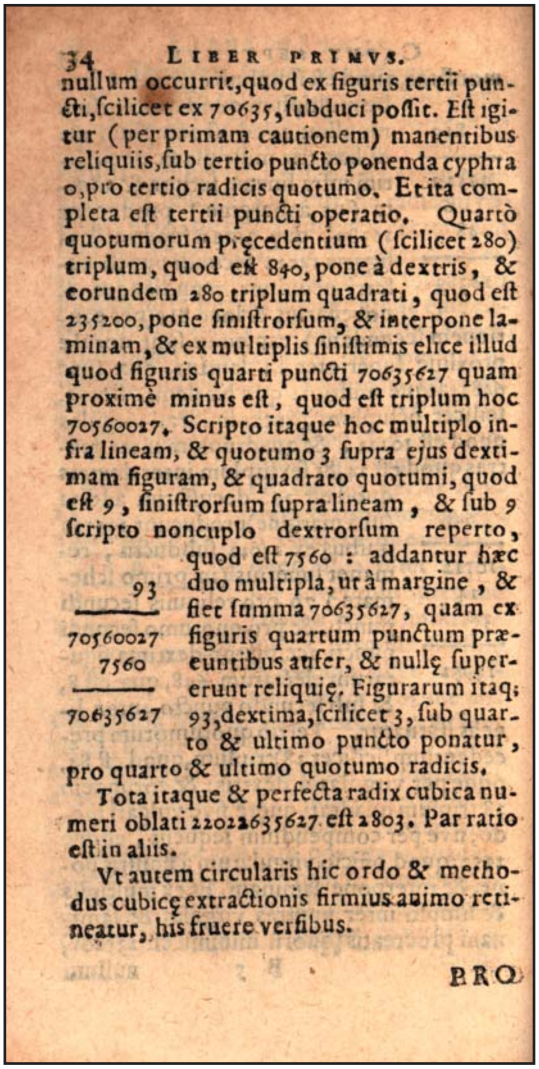

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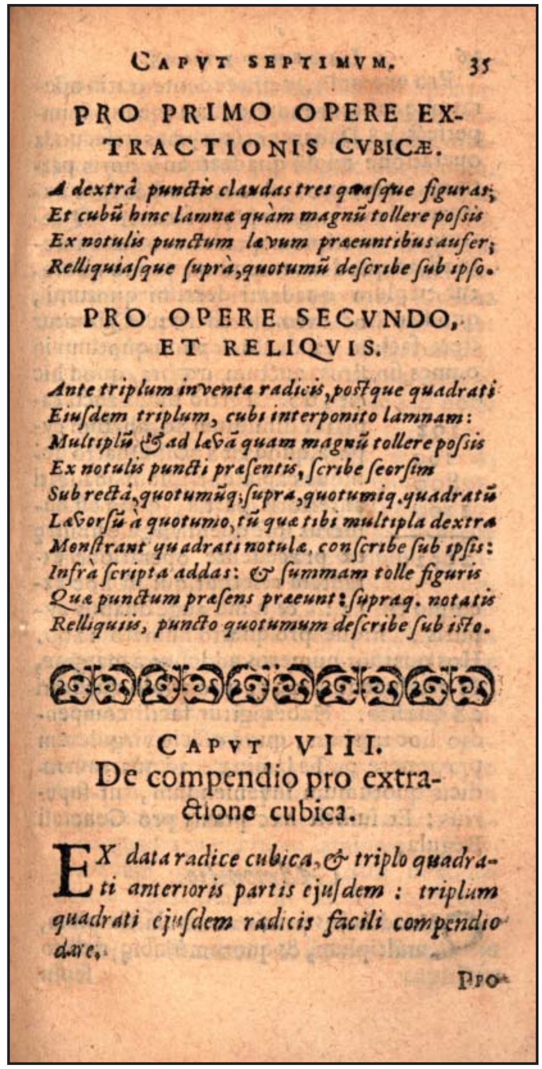
nullum occurrit, quod ex figuris tertii puncti, scilicet ex 70635, subduci possit. Est igitur (per primam cautionem) manentibus reliquiis, sub tertio puncto ponenda cyphra o, pro tertio radicis quotumo. Et ita completa est tertii puncti operatio. Quartò quotumorum pręcedentium (scilicet 280) triplum, quod est 840, pone à dextris, & eorundem 280 triplum quadrati, quod est 235200, pone sinistrorsum, & interpone laminam, & ex multiplis sinistimis elice illud quod figuris quarti puncti 70635627 quam proximè minus est, quod est triplum hoc 70560027. Scripto itaque hoc multiplo infra lineam, & quotumo 3 supra ejus dextimam figuram, & quadrato quotumi, quod est 9, sinistrorsum supra lineam, & sub 9 scripto noncuplo dextrorsum reperto, quod est 7560: addantur hæc duo multipla, ut à margine, & fiet summa 70635627, quam ex figuris quartum punctum præeuntibus aufer, & nullę supererunt reliquię. Figurarum itaq; 93, dextima, scilicet 3, sub quarto & ultimo puncto ponatur, pro quarto & ultimo quotumo radicis.

93
70560027
7560
70635627

Tota itaque & perfecta radix cubica numeri oblati 22022635627 est 2803. Par ratio est in aliis.

Vt autem circularis hic ordo & methodus cubicę extractionis firmius animo retineatur, his fruere versibus.

PRO



CAPVT SEPTIMVM. 35

PRO PRIMO OPERE EXTRACTIONIS CVBICÆ.

A dextrà punctis claudas tres quasque figuras;
Et cubũ hinc lamna quàm magnũ tollere possis
Ex notulis punctum laevum praeuntibus aufer;
Relliquiasque suprà, quotumũ describe sub ipso.

PRO OPERE SECVNDO, ET RELIQVIS.

Ante triplum inventa radicis, postque quadrati
Eiusdem triplum, cubi interponito lamnam:
Multiplũ & ad laevã quam magnũ tollere possis
Ex notulis puncti praesentis, scribe seorsim
Sub rectà, quotumũq; suprà, quotumiq, quadratũ
Laevorsũ à quotumo, tũ quae tibi multipla dextra
Monstrant quadrati notulae, conscribe sub ipsis:
Infrà scripta addas: & summam tolle figuris
Quae punctum praesens praeunt: supraq. notatis
Relliquiis, puncto quotumum describe sub isto.

CAPVT VIII.

De compendio pro extractione cubica.

EX data radice cubica, & triplo quadrati anterioris partis ejusdem: triplum quadrati ejusdem radicis facili compendio dare.

Pro-

Chapter VIII: A short cut method for finding the cube root.

This short chapter details a method of finding three times a number when the root is known and a similar method for determining three times the square from only a partially completed cube root operation. Both of these could be used when finding cube roots via the use of the cube root rod but, unless one were well versed in the operation, it would seem to be more confusing that helpful.

Pro exemplo, in præcedente tertia operatione dabatur radix cubica (quamvis imperfecta) 28. Dabatur etiam prius in secunda operatione triplũ quadrati anterioris partis ejusdem, quod est 12, quod & ipsæ extantes à læva virgulæ præ se ferunt. Quæritur autem triplum quadrati totius numeri 28, ad quod inveniendum primò quæratur triplum quadrati dextimi quotumi, quod in hoc exemplo est 192. Quæratur item factum ex ductu dextimi quotumi in omnes sinistros, auctum cyphra, quod hîc est 160. Tertiò hujus aucti capiatur dimidium 80, auctum cyphra, quod est 800. Quartò denique capiatur triplum quadrati anterioris partis, quod est numerus ipse quem virgulæ sinistræ ex præcedente operatione extantem referunt, qui in hoc exemplo est 12: & hunc auge duabus cyphris, fitque pro quarto numero 1200. Hos quatuor numeros adde, ut à margine, & producentur 2352 pro triplo quadrati 28 quæsito. Habes igitur facili compendio hoc triplum, quod officio virgularum præponere possis laminæ, ad quartum radicis quotumum inveniendum, ut superiùs: Et sufficit hæc praxis pro Generali Regula.

192
160
800
1200
2352

Admonitio.

Quoad hujus praxis vocabula, simplum, multiplum, & quotumũ, ubiq; debito sensu

sensu

sensu capimus; scilicet, simplum, pro eo quod ductum in quotumum producit multiplum. Multiplum, pro eo, quod divisum per simplum, producit quotumum. Quotumum vocamus, qui ductus in simplum producit multiplum, aut qui oritur ex divisione multipli per suum simplum. Multipla etiam & quotumi (quorum frequentior est usus in hac epitome) loca sua constanter in omni operatione retinent: ut duplum secundum areæ intervallum, triplum tertium, quadruplum quartum, & sic deinceps ad noncuplum quod in nono intervallo reperies.

Eorundem autem quotumi 2, 3, 4, 5, &c. usque ad 9 tam sub numeris ordinis intervallorum tacitè, quam suis locis in dextima laminæ columna expressè continentur. In situ autem simpli discrimen solum est, ejus enim figuræ dextimæ, unius vel duarum, locus semper variatur pro diversitate operis. Nonnunquam enim omnes tam dextimæ quam sinistimæ figuræ simpli reperiuntur in capitibus suarum virgularum, ut in multiplicatione & divisione. Nonnunquam unica tantum dextima figura in eodem intervallo tertiæ columnæ, quo suum multiplum reperitur; & cæteræ in capitibus virgularum, ut in extractione radicis quadratæ per suam laminam. Nonnunquam denique ejus duæ dextimæ figuræ reperiuntur in mediæ columnæ intervallo eodem quo suum multiplum, & cæteræ figuræ simpli in capitibus

capitibus

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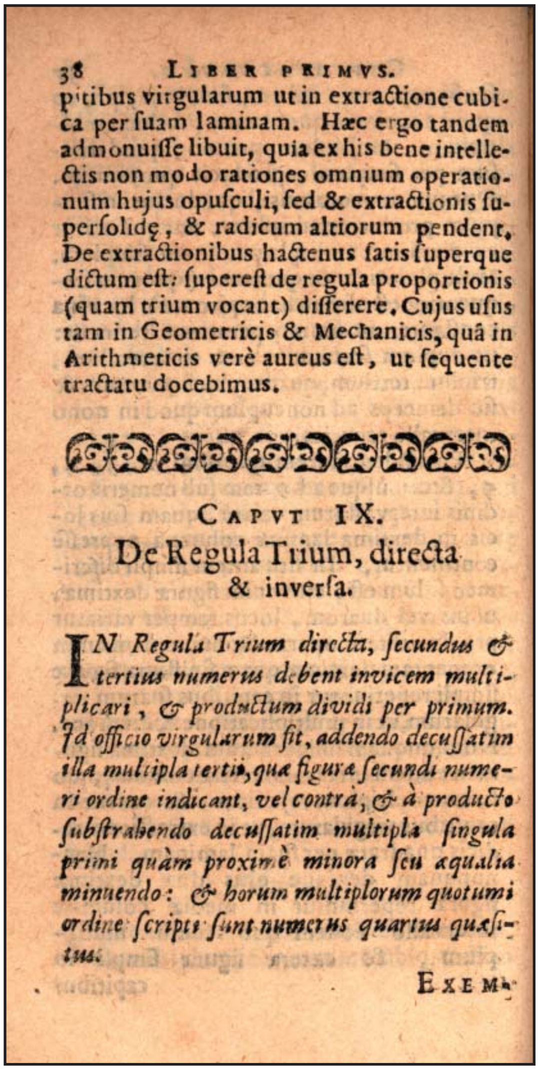

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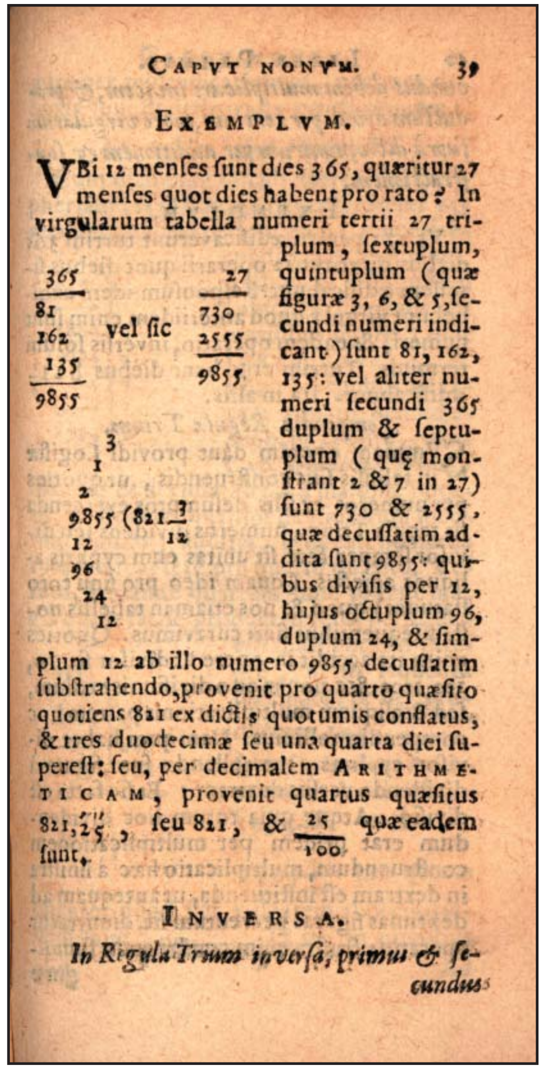
pitibus virgularum ut in extractione cubica per ſuam laminam. Hæc ergo tandem admonuiſſe libuit, quia ex his bene intellectis non modo rationes omnium operationum hujus opuſculi, ſed & extractionis ſuperſolidę, & radicum altiorum pendent. De extractionibus hactenus ſatis ſuperque dictum eſt: ſupereſt de regula proportionis (quam trium vocant) diſſerere. Cujus uſus tam in Geometricis & Mechanicis, quâ in Arithmeticis verè aureus eſt, ut ſequente tractatu docebimus.

CAPVT IX.

De Regula Trium, directa & inverſa.

IN Regula Trium directa, ſecundus & tertius numerus debent invicem multiplicari, & productum dividi per primum. Id officio virgularum fit, addendo decuſſatim illa multipla tertii, quæ figuræ ſecundi numeri ordine indicant, vel contra, & à producto ſubſtrahendo decuſſatim multipla ſingula primi quàm proximè minora ſeu æqualia minuendo: & horum multiplorum quotumi ordine ſcripti ſunt numerus quartus quæſitus.

EXEM-



CAPVT NONVM. 39

EXEMPLVM.

VBi 12 menſes ſunt dies 365, quæritur 27 menſes quot dies habent pro rato? In virgularum tabella numeri tertii 27 triplum, ſextuplum, quintuplum (quæ figuræ 3, 6, & 5, ſecundi numeri indicant) ſunt 81, 162, 135: vel aliter numeri ſecundi 365 duplum & ſeptuplum (quę monſtrant 2 & 7 in 27) ſunt 730 & 2555, quæ decuſſatim addita ſunt 9855: quibus diviſis per 12, hujus octuplum 96, duplum 24, & ſimplum 12 ab illo numero 9855 decuſſatim ſubſtrahendo, provenit pro quarto quæſito quotiens 821 ex dictis quotumis conflatus, & tres duodecimæ ſeu una quarta diei ſupereſt: ſeu, per decimalem ARITHMETICAM, provenit quartus quæſitus 821,2'5'', ſeu 821, & $\frac{25}{100}$ quæ eadem ſunt.

365 | 81 | 162 | 135 | 9855 — vel ſic — 27 | 730 | 2555 | 9855

9855 (821 $\frac{3}{12}$ — 12 | 96 | 24 | 12

INVERSA.

In Regula Trium inverſa, primus & ſecundus

Chapter IX: The rule of three, direct and inverse.

The *rule of three*, often called the *golden rule*, was a standard method of stating and solving problems. It was a regular section in almost every arithmetic book printed until modern times. Essentially it solves problems of the type: if 3 carrots cost 10 cents, how much to 27 carrots cost? It obviously gets it’s name from the fact that three numbers in some relationship are given and a fourth number is sought.

The above example of the cost of carrots is known as the *direct rule of three* and the numbers are always referred to as the first, second, and third numbers when describing the process of finding the fourth. For example, a typical algorithm would be stated as: *the second and third numbers must be multiplied together and then divided by the first to obtain the fourth*.

The *inverse rule of three* would be the same problem but with inverse ratios in most cases. It would typically be used for problems such as: if 17 workmen could dig a trench in 12 days, how many days would 5 workmen take? The solution would be expressed as *multiply the first and second numbers together and divide the result by the third.*

Napier provides an elementary example of each form. The first states that if 12 months contain 365 days then how many days are in 27 months. The second is if 27 workers construct a tower in 365 days, how long will 12 workers require to do the same job?

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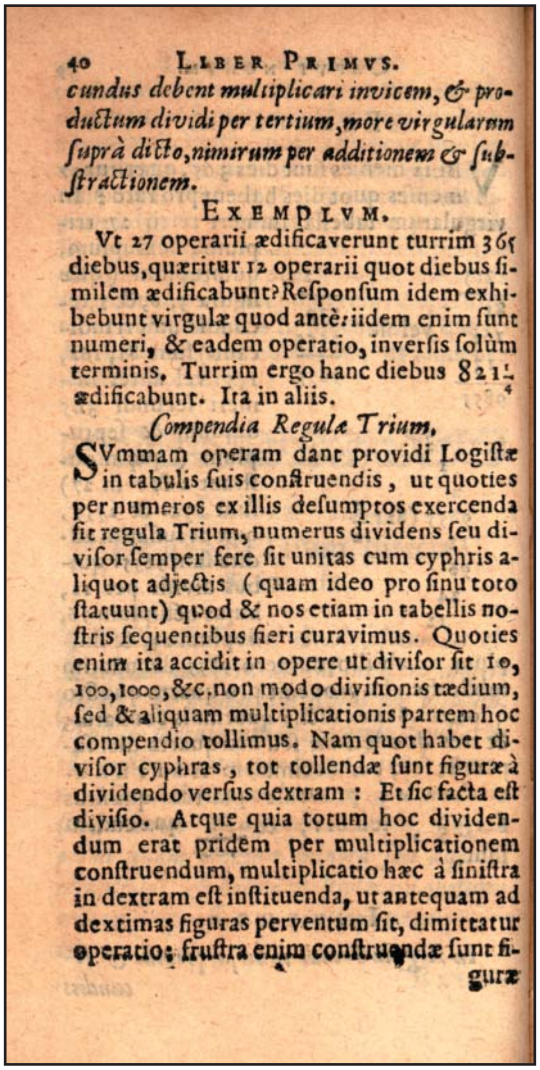

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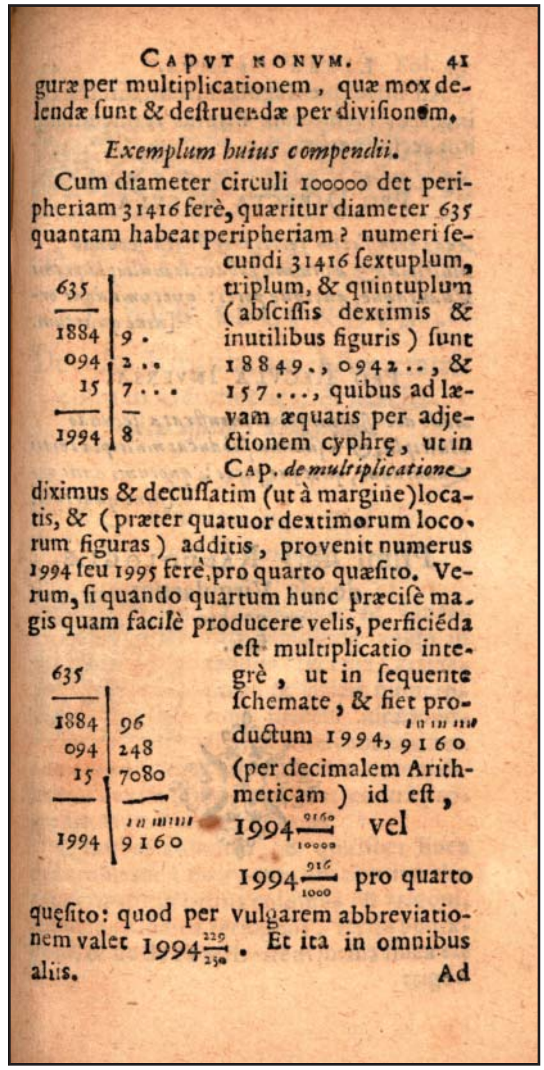
cundus debent multiplicari invicem, & productum dividi per tertium, more virgularum suprà dicto, nimirum per additionem & substractionem.

EXEMPLVM.

Vt 27 operarii ædificaverunt turrim 365 diebus, quæritur 12 operarii quot diebus similem ædificabunt? Responsum idem exhibebunt virgulæ quod antè: iidem enim sunt numeri, & eadem operatio, inversis solùm terminis. Turrim ergo hanc diebus $821\frac{1}{4}$ ædificabunt. Ita in aliis.

Compendia Regulæ Trium.

SVmmam operam dant providi Logistæ in tabulis suis construendis, ut quoties per numeros ex illis desumptos exercenda sit regula Trium, numerus dividens seu divisor semper fere sit unitas cum cyphris aliquot adjectis (quam ideo pro sinu toto statuunt) quod & nos etiam in tabellis nostris sequentibus fieri curavimus. Quoties enim ita accidit in opere ut divisor sit 10, 100, 1000, &c. non modo divisionis tædium, sed & aliquam multiplicationis partem hoc compendio tollimus. Nam quot habet divisor cyphras, tot tollendæ sunt figuræ à dividendo versus dextram: Et sic facta est divisio. Atque quia totum hoc dividendum erat pridem per multiplicationem construendum, multiplicatio hæc à sinistra in dextram est instituenda, ut antequam ad dextimas figuras perventum sit, dimittatur operatio: frustra enim construendæ sunt figuræ



CAPVT NONVM. 41

guræ per multiplicationem, quæ mox delendæ sunt & destruendæ per divisionem.

Exemplum huius compendii.

Cum diameter circuli 100000 det peripheriam 31416 ferè, quæritur diameter 635 quantam habeat peripheriam? numeri secundi 31416 sextuplum, triplum, & quintuplum (abscissis dextimis & inutilibus figuris) sunt 18849., 0942.., & 157..., quibus ad lævam æquatis per adjectionem cyphræ, ut in CAP. *de multiplicatione* diximus & decussatim (ut à margine) locatis, & (præter quatuor dextimorum locorum figuras) additis, provenit numerus 1994 seu 1995 ferè, pro quarto quæsito. Verum, si quando quartum hunc præcisè magis quam facilè producere velis, perficiéda est multiplicatio integrè, ut in sequente schemate, & fiet productum 1994, 9160 (per decimalem Arithmeticam) id est, $1994\frac{9160}{10000}$ vel $1994\frac{916}{1000}$ pro quarto quæsito: quod per vulgarem abbreviationem valet $1994\frac{229}{250}$. Et ita in omnibus aliis.

635	
1884	9.
094	2..
15	7...
1994	8

635	
1884	96
094	248
15	7080
1994	9160

Ad

The section titled *Compendia Regula Trium* simply explains that if one of the terms in the rule of three is a power of 10 (although Napier does not use that term–he actually gives examples such as 10, 100, 1000 etc.), then the multiplication or division by those numbers is simply the addition or removal of zeros (i.e., shifting the decimal point). While this point is obvious to us now, it must be remembered that this was written at the very birth of decimal fractions and such operations would not have been known to most readers of this book.

Napier observes that many problems and tables contain numbers such as 10, 100 and 1000, thus this short cut method is often very useful. He goes on to prove his point because all the tables and problems in the following *Second Book* contain such numbers.

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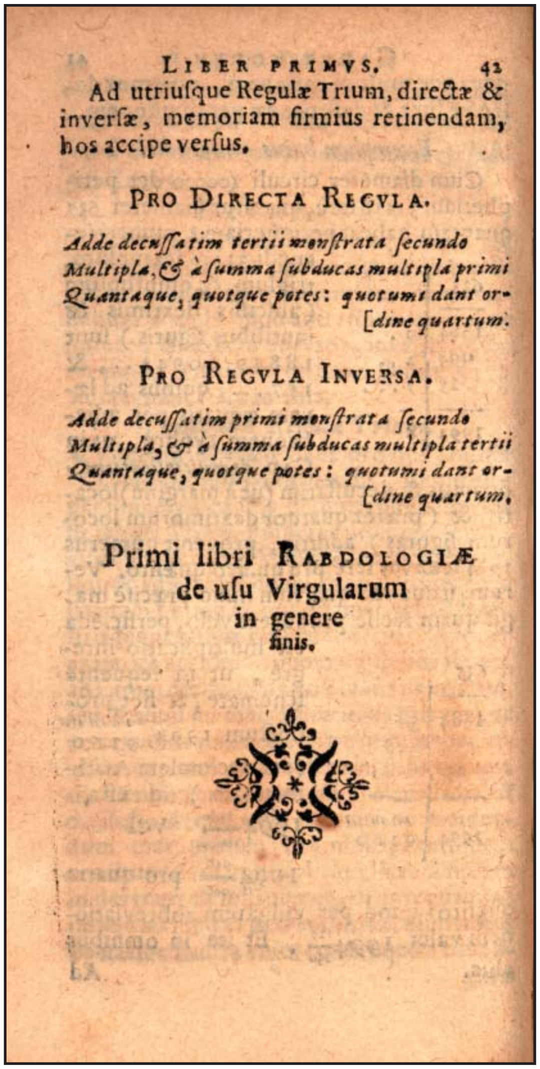


Ad utriuſque Regulæ Trium, directæ & inverſæ, memoriam firmius retinendam, hos accipe verſus.

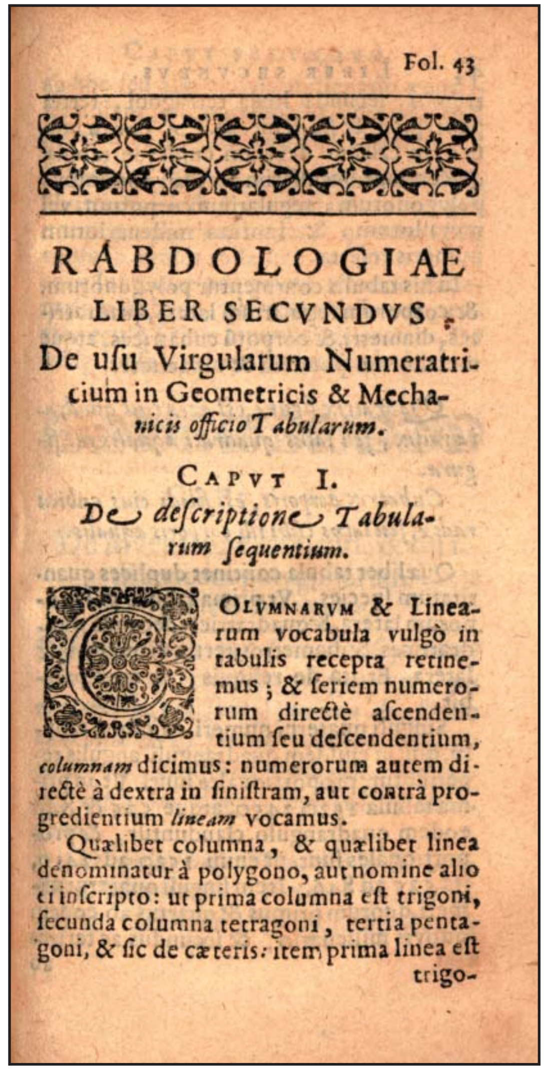
PRO DIRECTA REGVLA.

Adde decuſſatim tertii monſtrata ſecundo
Multipla, & à ſumma ſubducas multipla primi
Quantaque, quotque potes: quotumi dant ordine quartum.

PRO REGVLA INVERSA.

Adde decuſſatim primi monſtrata ſecundo
Multipla, & à ſumma ſubducas multipla tertii
Quantaque, quotque potes: quotumi dant ordine quartum.

Primi libri RABDOLOGIÆ de uſu Virgularum in genere finis.



RABDOLOGIAE LIBER SECVNDVS.

De uſu Virgularum Numeratricium in Geometricis & Mechanicis *officio Tabularum.*

CAPVT I.

De deſcriptione Tabularum ſequentium.

COLVMNARVM & Linearum vocabula vulgò in tabulis recepta retinemus; & ſeriem numerorum directè aſcendentium ſeu deſcendentium, *columnam* dicimus: numerorum autem directè à dextra in ſiniſtram, aut contrà progredientium *lineam* vocamus.

Quælibet columna, & quælibet linea denominatur à polygono, aut nomine alio ei inſcripto: ut prima columna eſt trigoni, ſecunda columna tetragoni, tertia pentagoni, & ſic de cæteris: item prima linea eſt trigo-

Second book: The use of the rods in geometry and numerical problems.

Chapter I: Description of the tables.

Napier uses several tables to give physical constants about regular figures and density of metals which he puts to use in the following problems. Knowing the density of various metals was useful in gunnery where the weight of the shot often dictated the amount of gun powder used, but these also had a commercial use.

The first tables are found on double pages 48—49, 54—55 and 62—63 and several smaller tables follow later in the chapter.

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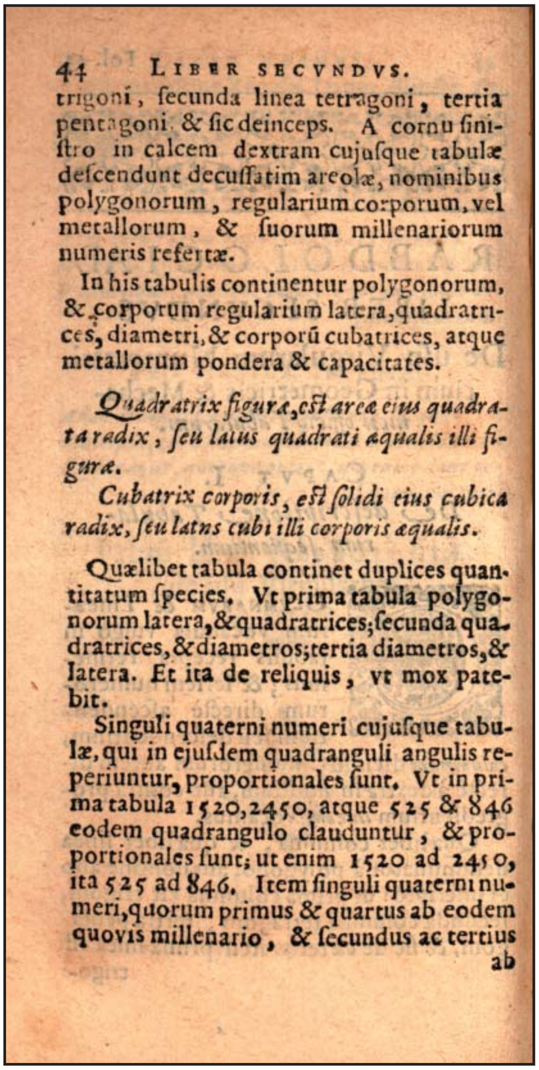

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trigoni, ſecunda linea tetragoni, tertia pentagoni & ſic deinceps. A cornu ſiniſtro in calcem dextram cujuſque tabulæ deſcendunt decuſſatim areolæ, nominibus polygonorum, regularium corporum, vel metallorum, & ſuorum millenariorum numeris refertæ.

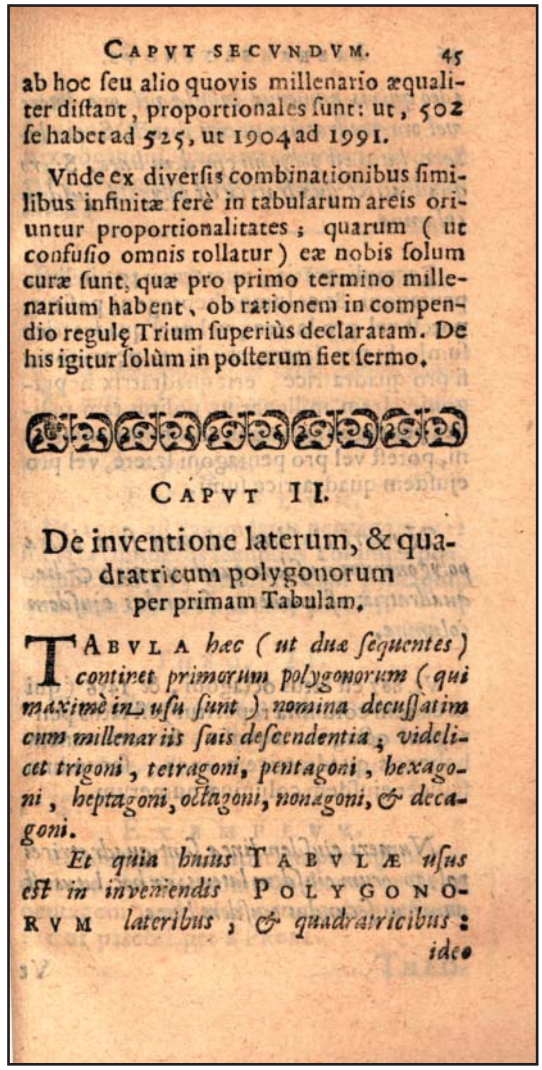
In his tabulis continentur polygonorum, & corporum regularium latera, quadratrices, diametri, & corporũ cubatrices, atque metallorum pondera & capacitates.

Quadratrix figuræ, eſt area eius quadrata radix, ſeu latus quadrati æqualis illi figuræ.

Cubatrix corporis, eſt ſolidi eius cubica radix, ſeu latus cubi illi corporis æqualis.

Quælibet tabula continet duplices quantitatum ſpecies. Vt prima tabula polygonorum latera, & quadratrices; ſecunda quadratrices, & diametros; tertia diametros, & latera. Et ita de reliquis, vt mox patebit.

Singuli quaterni numeri cujuſque tabulæ, qui in ejuſdem quadranguli angulis reperiuntur, proportionales ſunt. Vt in prima tabula 1520, 2450, atque 525 & 846 eodem quadrangulo clauduntur, & proportionales ſunt; ut enim 1520 ad 2450, ita 525 ad 846. Item ſinguli quaterni numeri, quorum primus & quartus ab eodem quovis millenario, & ſecundus ac tertius ab



CAPVT SECVNDVM. 45

ab hoc ſeu alio quovis millenario æqualiter diſtant, proportionales ſunt: ut, 502 ſe habet ad 525, ut 1904 ad 1991.

Vnde ex diverſis combinationibus ſimilibus infinitæ ferè in tabularum areis oriuntur proportionalitates; quarum (ut confuſio omnis tollatur) eæ nobis ſolum curæ ſunt, quæ pro primo termino millenarium habent, ob rationem in compendio regulę Trium ſuperiùs declaratam. De his igitur ſolùm in poſterum fiet ſermo.

CAPVT II.

De inventione laterum, & quadratricum polygonorum per primam Tabulam.

TABVLA *hæc (ut duæ ſequentes) continet primorum polygonorum (qui maximè in uſu ſunt) nomina decuſſatim cum millenariis ſuis deſcendentia; videlicet trigoni, tetragoni, pentagoni, hexagoni, heptagoni, octagoni, nonagoni, & decagoni.*

Et quia huius TABVLÆ *uſus eſt in inveniendis* POLYGONORVM *lateribus, & quadratricibus:* ideo

The diagonal elements of the first table contain names of regular geometric figures (triangle, square, pentagon, ... decagon) and these are the labels for both the rows and the columns. The diagonal elements also contain the number 1000, which is considered to be the defining number for each figure.

Napier was fascinated with ratios and these tables contain ratios of lengths of sides, areas, sides of squares that equal the area of a particular figure, volumes and diameters of solids and similar information. They are not easy to use as some of the ratios are looked up by noting the position of various numbers being an equal distance from the diagonal (horizontal or vertical) and some are always in the second position of any given row or column. The problems are all similar to: given the side of a square of a particular area, find the length of the side of a heptagon of equal area.

ideo quivis numerus tabulæ vel pro latere vel pro quadratrice accipi potest. Si pro latere, latus est polygoni ejusdem lineæ. Si pro quadratrice, quadratrix est polygoni ejusdem columnæ.

Exempli gratia, numerus 1456 in linea pentagoni, & columna heptagoni positus, potest vel pro latere, vel pro quadratrice sumi. Si pro latere, erit latus pentagoni: si pro quadratrice, erit quadratrix heptagoni. Item millenarius positus tam in linea pentagoni, quam in columna pentagoni, potest vel pro pentagoni latere, vel pro ejusdem quadratrice sumi.

Numeri ejusdem columnæ sunt latera polygonorum ejusdem quadratricis: & hæc quadratrix est numerus secundus ejusdem columnæ.

Vt, 867 est latus octagoni, & 1456 (qui in eadem columna reperitur) est latus pentagoni octagono æqualis, & communem habentis quadratricem 1904, secundum scilicet ejusdem columnæ numerum.

Numeri eiusdem lineæ sunt quadratrices polygonorum eiusdem lateris: & hoc latus est numerus secundus eiusdem lineæ.

Vt

Vt 687 est quadratrix pentagoni, & 1301 (qui in eadem linea reperitur) est quadratrix nonagoni, quorum commune latus est 525, secundus scilicet numerus ejusdem lineæ.

Præcipua analogorum Theoremata.

THEOR. I.

VT millenarius ad latus datum nominati polygoni: ita numerus secundus columnæ nominati polygoni, ad quadratricem eiusdem polygoni.

EXEMPLVM.

Vt 1000 ad datum latus pentagoni 315: ita 1312 (numerus secundus columnæ pentagoni) ad 413 quadratricem pentagoni quæsitam: ut ex PROBL. 1. patebit.

THEO. II.

Vt millenarius ad quadratricem datam alicujus nominati polygoni: ita numerus secundus lineæ illius polygoni ad latus eiusdem polygoni.

EXEMPLVM.

Vt 1000 ad quadratricem pentagoni datam 413: ita 762 (numerus secundus lineæ pentagoni) ad latus pentagoni quæsitum 315, ut patebit per 2 PROBL.

THEO-

THEOR. III.

Duorum polygonorum æqualium seu eiusdem quadratricis, ut millenarius ad latus datum primi; ita numerus interceptus à columna primi & linea secundi ad latus secundi.

EXEMPLVM.

Sint æqualia polygona pentagonum cujus latus sit 315, & trigonum cujus latus quæritur. Erit ut 1000 ad latus datum 315, ita 1991 (numerus interceptus à columna pentagoni, & linea trigoni) ad quæsitum latus trigoni quod est 627, ut infrà problem. 3 patebit.

Tabella prima laterum & quadratricum polygonorum.

Trigoni 1000	1520	1991	2450	2896	3344	3771	4217
658	*Tetragoni* 1000	1312	1612	1904	2196	2487	2769
502	762	*Pentagoni* 1000	1231	1456	2019	1895	2119
408	620	812	*Hexagoni* 1000	1182	1364	1539	1721
345	525	687	846	*Heptagoni.* 1000	1154	1301	1455
299	455	495	733	867	*Octagoni.* 1000	1128	1261
265	402	528	650	769	887	*Nonagoni.* 1000	1118
237	361	472	581	687	793	895	*Decagoni.* 1000

This is the table for regular polygons with 3 to 10 sides.

THEOR. IV.

Duorum polygonorum ejusdem lateris, ut millenarius ad quadratricem primi datam: ita numerus interceptus à linea primi & columna secundi, ad quadratricem secundi.

EXEMPLVM.

Sint polygona ejusdem lateris pentagonum cujus quadratrix sit 413, & trigonum cujus quadratrix quæritur. Erit ut 1000 ad 413 quadratricem datam, ita 502 (numerus interceptus à linea pentagoni & columna trigoni) ad quadratricem trigoni quæsitam 207. ut inferius problemate quarto patebit.

PROBLEMATA VSVS PRÆCEDENTIVM.

PROBL. I.

DAto latere polygoni nominati, dare ejusdem quadratricem.

EXEMPLVM.

SIt latus pentagoni 315. Ex theoremate primo erunt ut 1000 ad 315, ita 1312 (numerus secundus columnæ pentagoni) ad quadratricem pentagoni quæsitam. Et per compendium regulæ Trium, triplum, simplum, & quintuplum numeri 1312, vel simplum, triplum, simplum & duplum numeri 315 addita decussatim, & à producto abscissis

abscissis tribus dextimis figuris, producent 413 quadratricem pentagoni quæsitam, cujus latus est 315.

PROBL. II.

Datâ quadratrice polygoni nominati dare eiusdem latus.

EXEMPLVM.

Sit quadratrix pentagoni data 413, per 2 theorema erit ut 1000 ad 413 numerum datum; ita 762 (numerus secundus lineæ pentagoni) ad latus quæsitum. Abscinde ergo tres figuras à producto, quod fit ex septuplo, sextuplo, & duplo numeri 413; vel ex quadruplo, simplo, & triplo numeri 762 decussatim additis, & provenient 315 latus quæsitum pentagoni, cuius quadratrix data erat 413.

PROBL. III.

Duorum polygonorum æqualium seu eiusdem quadratricis, dato latere primi dare latus secundi, & utriusque quadratricem.

EXEMPLVM.

Sint æqualia seu ejusdem quadratricis pentagonum cujus latus sit 315, & trigonum cuius latus quæritur. Et quum per 3 theorema sit ut 1000 ad 315, ita 1991 (numerus interceptus à columna pentagoni & linea trigoni) ad quæsitum latus trigoni. Ideo (per cōpendium nostrū) abscinde tres dextimas figuras à producto, quod fit ex triplo, simplo, & quintuplo numeri 1991: vel quod fit ex simplo, noncuplo, & noncuplo ac simplo numeri 315 decussatim additis, & proveniet latus trigoni 627 quęsitum: quadratricem autem dabit problema primum, scilicet 413.

Probl. IV.

Duorum polygonorum quorum latera sunt æqualia, datâ quadratrice primi, dare quadratricem secundi, & utriusque latus.

Repete EXEMPLVM quarti THEOREMATIS, in quo Pentagonum, cujus quadratrix est 413, & Trigonum quæsitę quadratricis sunt æqualium laterum. Per illud enim theorema ut 1000 se habet ad 413 quadratricem pentagoni: ita 502 (numerus interceptus à linea pentagoni & columna trigoni) ad quadratricem trigoni quæsitam. Vnde (per compendium nostrum) abscissis tribus dextimis figuris à quadruplo, simplo, & triplo numeri 502: vel à quintuplo, cyphra, & duplo numeri 413 decussatim additis, provenient 207 pro quadratrice trigoni quæsita. Vtriusque autem latus dabit problema secundū, scilicet 315.

CAPVT III.

De inventione quadratricum & diametrorum polygonorum per Tabulam secundam.

HABET hęc Tabula (pręter communia) polygonorum quadratrices, & diametros: quas quia & circuli habent, circulum igitur inter hujus tabellę polygona numeramus tanquam polygonum infinitorum laterum. Per polygona igitur intellige etiam circulum, & per diametros polygonorum, intellige circuli diametrum, & reliquorum polygonorum diametrum maiorem, id est, diametrum circuli polygono circumscripti. Diametros enim minores circulorum polygonis inscriptorum tanquam minùs utiles missas facimus: earum enim præcipuo munere funguntur quadratrices.

Omnis itaque numerus huius Tabellæ vel pro quadratrice, vel pro diametro alicuius polygoni accipi potest. Si pro quadratrice, dicetur quadratrix polygoni eiusdem lineæ: si verò pro diametro sumatur, dicetur diameter polygoni eiusdem columnæ.

Numeri eiusdem columnæ, sunt quadratrices polygonorum eiusdem diametri: & hæc diameter est numerus infimus eiusdem columnæ.

Numeri eiusdem lineæ sunt diametri polygonorum eiusdem quadratricis: & hæc quadratrix est dextimus numerus eiusdem lineæ.

Tabella secunda quadratricum polygonorum & diametrorum circulorum iis circumscriptorum.

Trigoni 1000	806	739	707	689	678	670	665	643	570
1241	Tetrag. 1000	917	877	855	841	831	825	798	707
1353	1090	Pentag. 1000	957	932	917	907	900	870	771
1414	1140	1045	Hexag. 1000	974	959	947	940	909	806
1451	1169	1073	1026	Heptag. 1000	984	972	965	933	827
1476	1188	1090	1043	1016	Octagon. 1000	989	982	950	841
1492	1203	1103	1056	1029	1011	Nonago. 1000	992	959	850
1504	1212	1112	1063	1036	1019	1008	decago. 1000	967	857
1555	1253	1149	1100	1072	1054	1042	1034	Circuli 1000	886
1755	1414	1297	1240	1209	1189	1176	1167	1128	1000

This table gives areas of various polygons and diameters of their circumscribing circles.

Præcipua Analoga secundæ TABULÆ.

Theor. 1.

VT millenarius ad quadratricem nominati polygoni datam : ita numerus infimus columnę illius polygoni, ad eiusdem polygoni diametrum.

Theor. 2.

Ut millenarius ad diametrum nominati polygoni datam : ita numerus dextimus lineæ illius polygoni, ad eiusdem polygoni quadratricem.

Theor. 3.

Duorum polygonorum eiusdem diametri ut millenarius ad quadratricem primi : ita numerus interceptus à columna primi & linea secundi ad quadratricem secundi.

Theor. 4.

Duorum polygonorum eiusdem quadratricis ut millenarius ad diametrum primi: ita numerus interceptus à linea primi & columna secundi ad diametrum secundi.

Admonitio.

Quia præmissa partim ex simili doctrina primę tabulæ, partim per exempla sequentium problematum satis perspicua sunt, & propriis suis exemplis non egent: pro exemplis igitur theorematum præcedentium, exempla sequentium suorum problematum accipe.

Problemata usus secundæ TABULÆ.

Prob. 1.

DAtâ quadratrice nominati polygoni, eiusdem polygoni diametrum dare.

Exemplum.

Detur quadratrix trigoni 800, quæritur ejusdem trigoni diameter seu diameter circuli circumscripti huic trigono. Per primum theorema erit ut 1000 ad 800 quadratricem trigoni datam : ita 1755, (numerus infimus columnæ trigoni) ad ejusdem diametrum quęsitam. Ergo (per compendium nostrum, regulę Trium) junge decussatim octuplum numeri 1755, & duas cyphras (propter numerum datum 800) & fient 1404000, à quo numero ablatis tribus dextimis figuris restant 1404 pro diametro trigoni quæsita, cujus quadratrix dabatur 800.

Probl. 2.

Datâ diametro polygoni nominati, ejusdem quadratricem dare.

EXEMPLVM.

Detur diameter trigoni 1404, & quæratur ejusdem trigoni quadratrix. Per 2 theorema erit, ut 1000 ad 1404 diametrum trigoni datam; sic 570 (numerus dextimus lineę trigoni) ad quadratricem eiusdem trigoni quęsitam. Adde ergo quintuplum, septuplum, & cyphram numeri 1404, vel simplum, quadruplum, cyphram, & quadruplum numeri 570 decussatim, & fient 800280, quarum abscissis tribus dextimis figuris, supersunt 800 pro quadratrice trigoni quęsita, cujus diameter dabatur 1404.

PROBL. III.

Duorum polygonorum eiusdem diametri datâ quadratrice primi, quadratricem secundi dare, & utriusque diametrum.

Exemplum.

Sint duo polygona eiusdem diametri, primum circulus cuius quadratrix data sit 1205, & secundum sit heptagonum, cujus quæritur quadratrix. Per 3 theorema erit ut millenarius ad 1205 quadratricem circuli datam; ita 933 (numerus interceptus à columna circuli & linea heptagoni) ad quadratricem heptagoni quæsitam. Adde ergo decussatim noncuplum, triplum, & triplum numeri 1205, vel simplum, duplum,

plum, cyphram, & quintuplum numeri 993, & fient 1124265, quarum abscissis tribus dextimis figuris, restant 1124 pro quadratrice heptagoni quęsita. Diametrum autem communem circuli & heptagoni per 1 Probl. venari poteris si libet, estque 1359 ferè.

Probl. IIII.

Duorum polygonorum eiusdem quadratricis datâ diametro primi, diametrum secundi & utriusque quadratricem notas reddere.

Exemplum.

Sint polygona, primum nonagonum, secundum circulus, æqualia seu ejusdem quadratricis, deturque diameter nonagoni 1302, quæritur autem circuli diameter. Per 4 theorema ut se habet 1000 ad 1302 diametrum nonagoni datam; ita se habebit 959 (numerus interceptus à linea nonagoni & columna circuli) ad diametrum circuli quæsitam. Adde ergo decussatim noncuplum, quintuplum, & noncuplum numeri 1302; vel simplum, triplum, cyphram, & duplum numeri 959, & fient 1248618, quarum deletis tribus dextimis figuris, remanent 1249 ferè pro diametro circuli quæsita. Communem autem nonagoni & circuli quadratricem, si libet, per 2 Probl. acquirere poteris, estque 1107 ferè.

CAPVT IV.

De inventione diametrorum & laterum polygonorum per tertiam Tabulam.

Continet hæc tertia Tabula polygonorum diametros & latera, eorumque ad invicem proportiones. Omnis itaque numerus huius tabulæ vel pro diametro, vel pro latere alicujus polygoni accipi potest. Si pro diametro, dicetur diameter polygoni eiusdem lineæ: si pro latere, dicetur latus polygoni eiusdem columnæ.

Numeri eiusdem columnæ sunt diametri polygonorum eiusdem lateris: & hoc latus est numerus supremus eiusdem columnæ.

Numeri eiusdem lineæ sunt latera polygonorum eiusdem diametri: & hæc diameter est primus numerus eiusdem lineæ.

Præcipua Analoga tertiæ TABULÆ.

Theor. 1.

Vt millenarius ad diametrum nominati polygoni datam: ita numerus supremus columnæ illius polygoni ad eiusdem polygoni latus.

Vt mil-

Theor. 2.

Vt millenarius ad datum latus nominati polygoni: ita numerus primus lineæ illius polygoni ad eiusdem polygoni diametrum.

Theor. 3.

Duorum polygonorum eiusdem lateris ut millenarius ad diametrum primi: ita numerus interceptus à columna primi & linea secundi ad diametrum secundi.

Theor. 4.

Duorum polygonorum eiusdem diametri ut millenarius ad latus primi: ita numerus interceptus à linea primi & columna secundi ad latus secundi.

Pro præcedentium exemplis accipe exempla problematum sequentium, quæ illorum etiam sunt.

Problemata usus tertiæ Tabulæ.

Prob. 1.

Datâ diametro majore nominati polygoni, latus ejusdem polygoni invenire.

Exemplum.

Sit diameter pentagoni data 536, & quæratur ejusdem latus. Per primum theorema erit ut 1000 ad 536 diametrum datam: ita 588 (numerus supremus columnæ pentagoni) ad eiusdem latus quæsitum. Ergo (per compendium regulæ TRIVM) quintuplo, octuplo, & octuplo numeri 536; vel quintuplo, triplo, & sextuplo numeri

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numeri 588 decussatim additis, provenient inde 315168, à quibus abstrahendæ sunt tres novissimę figuræ, & restant 315 pro latere pentagoni quæsito, cuius maior diameter dabatur 536.

Tabella tertia laterum polygonorum & diametrorum circulorum iis circumscriptorum.

1000	866	707	588	500	434	383	342	309
1154	*Trigoni* 1000	817	676	577	501	442	394	357
1414	1225	*Tetragoni* 1000	832	707	614	541	483	437
1700	1472	1202	*Pentagoni* 1000	850	738	650	580	525
2000	1732	1414	1176	*hexag.* 1000	868	765	684	618
2304	1995	1629	1355	1152	*Heptag.* 1000	881	786	712
2614	2264	1848	1537	1307	1134	*octagon.* 1000	891	807
2929	2537	2071	1722	1462	1271	1122	*Nonag.* 1000	904
3236	2802	2288	1903	1618	1404	1239	1107	*Decag.* 1000

Probl. II.

Dato latere nominati polygoni, diametrum einsdem maiorem reperire.

This third table provides information on the lengths of sides of polygons and their circumscribing circles.

Exemplum.

Sit latus pentagoni datum 315, & quæratur ejusdem diameter. Per 2 theorema, erit ut 1000 ad datum latus 315: ita 1700 (numerus primus lineę pentagoni) ad ejusdem diametrum quæsitam. Vnde triplum, simplum, & quintuplum numeri 1700; vel simplum, septuplum, cyphra, & cyphra numeri 315 decussatim addita, producunt 535500: quæ minuta tribus dextimis notis reddunt 536 ferè pro diametro pentagoni quæsita, cujus latus dabatur 315.

Probl. 3.

Duorum polygonorum einsdem lateris, datâ diametro primi, diametrum secundi, & utriusque latus commune invenire.

Exemplum.

Sint duo polygona ejusdem lateris, pentagonum primum, & trigonum secundum. Pentagoni detur diameter 536, trigoni verò diameter quæratur. Erit (per tertium theorema) ut millenarius ad 536 diametrum pentagoni datam: ita 679 (numerus interceptus à columna pentagoni & linea trigoni) ad diametrum trigoni quæsitam. Itaque quintuplum, triplum, & sextuplum numeri 679: vel sextuplum, septuplum, & noncuplum numeri 536. Addita

dita decussatim, & minuta tribus dextimis figuris producunt 364 ferè pro diametro trigoni quæsita. Si præterea latus commune utriusque quæsiveris, invenies illud per primum problema esse 315, ut suprà.

Probl. 4.

Duorum polygonorum eiusdem diametri, dato latere primi, latus secundi, & utriusque communem diametrum invenire.

Exemplum.

Sint pentagonum & trigonum ejusdem diametri: pentagoni pro primo detur latus 315, trigoni pro secundo quæratur latus. Per quartũ theorema erit ut 1000 ad 315 pentagoni latus datum: ita 1472 (numerus interceptus à linea pentagoni & columna trigoni) ad trigoni latus quæsitũ. Adde ergo decussatim, triplum, simplum, & quintuplum numeri 1472 (vel contrà illius pro hujus multipla) & provenient inde 463680, vnde abscissis tribus dextimis restant 464 ferè pro latere trigoni quæsito. Si præterea communem utriusque diametrum quæsiveris, eam per 2 problema invenies esse 536.

ADMONITIO.

IN numeri sunt alii harum & subsequentium Tabularum vsus, quorum quidam particularibus numeris propriè incidunt (ut numerum datum quàm proximè

ximè secare per extremam & mediam rationem virtute trium numerorum tertiæ tabulæ 618, 1000, & 1618.) *Quidam verò alii usus miscellanei sunt, & ex superioribus theorematibus componuntur (ut quatuor polygonorum, trigoni & pentagoni eiusdem lateris, pentagoni & heptagoni eiusdem quadratricis, heptagoni & nonagoni eiusdem diametri, dato unico cuiusvis latere, quadratrice, vel diametro, reliquas omnes reliquorum omnium dare.) Quos usus quivis ingenii mediocris per se intelliget ex præmissis: non enim omnes harum usus caperet hæc brevis epitome, nec in ea instituimus Arithmeticam, & Geometriam, sed virgularum tantum in iis usum docere.*

Hactenus latera, quadratrices, & diametros polygonorum invenire docuimus: superest de inventione laterum, cubatricum, & diametrorum corporum quinque regularium, & sphæra, sequentibus his tribus tabellis disserere.

CAPVT

CAPVT V.
De lateribus & cubatricibus quinque corporum regularium inveniendis per quartam TABVLAM.

QVarta Tabula (*quæ & prima Stereometricarum dimensionum est*) *continet latera & cubatrices quinque corporum regularium. Omnis itaque numerus hujus tabellæ vel pro latere, vel pro cubatrice alicujus corporis regularis accipi potest: si pro latere, dicetur latus corporis regularis ejusdem lineæ: si pro cubatrice, dicetur cubatrix corporis regularis eiusdem columnæ.*

Numeri eiusdem columnæ sunt latera corporum regularium eiusdem cubatricis: & hæc cubatrix est numerus medius eiusdem columnæ.

Numeri eiusdem lineæ sunt cubatrices corporum eiusdem lateris: & hoc latus est numerus medius eiusdem lineæ.

Præcipua analoga 4 Tabulæ.
THEOR. I.

VT *millenarius ad latus datum corporis regularis nominati: ita numerus* medius

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medius columnæ illius corporis ad eiusdem corporis regularis cubatricem.

Theor. II.

Ut millenarius ad cubatricem datam corporis regularis nominati: ita numerus medius lineæ illius corporis ad latus eiusdem corporis.

Theor. III.

Duorum corporum regularium æqualium seu eiusdem cubatricis ut millenarius ad latus datum primi: ita numerus interceptus à columna primi & linea secundi ad latus secundi.

Tabella quarta laterum & cubatricum quinque regularium corporum.

Tetrae. 1000	1587	2040	2689	4088
630	*Octaed.* 1000	1285	1694	2575
490	778	*Cubus.* 1000	1318	2003
372	590	759	*Icosaed.* 1000	1521
245	388	499	658	*Dodec.* 1000

THEOR. IV.

Duorum corporum regularium eiusdem lateris, ut millenarius ad cubatricem primi datam: ita numerus interceptus à linea primi & columna secundi ad cubatricem secundi.

PROBLEMATA usus quartæ TABVLÆ.

Prob. I.

DATO *latere corporis regularis nominati, eiusdem corporis cubatricem dare.*

Exemplum.

SIT Octaedri latus datum 452, ejusdem quæritur cubatrix. Per primum theorema ut se habet millenarius ad 452 latus Octaedri datum: ita 778 (numerus medius columnæ Octaedri) ad cubatricem ejusdem quæsitam. Vnde summa ex quadruplo, quintuplo, & duplo numeri 778: vel septuplo, septuplo, & octuplo numeri 452 additis decussatim, minuta tribus dextimis figuris, est 352 ferè, cubatrix scilicet petita Octaedri, cujus latus dabatur 452.

Prob.

The fourth table gives the lengths of the sides and volumes contained in regular solids.

PROBL. II.

Datâ cubatrice corporis regularis nominati, ejusdem corporis latus invenire.

EXEMPLVM.

Sit octaedri cubatrix 352 data, eiusdem latus quæritur. Per 2 theorema ut se habet millenarius ad 352 cubatricem octaedri datam: ita se habebit 1285 (numerus medius lineæ octaedri) ad eiusdem octaedri latus quęsitum. Vnde triplum, quintuplum, & duplum numeri 1285 (vel contrà illius pro hujus multipla) decussatim addita & minuta tribus dextimis notulis producunt 452 latus octaedri quæsitum, cuius scilicet cubatrix dabatur 352.

PROBL. III.

Duorum corporum regularium æqualium seu eiusdem cubatricis, dato latere primi, latus etiam secundi, & utriusque cubatricem communem invenire.

Exemplum.

Sint duo corpora ęqualia, octaedrum primum, & icosaedrum secundum: octaedri latus detur 452, icosaedri quæritur. Per 3 theorema vt se habet millenarius ad 452 latus octaedri datum: ita 590 (numerus interceptus à columna octaedri & linea icosaedri) ad latus icosaedri quæsitum. Vnde quadruplum, quintuplum, & duplum numeri 590: vel quintuplum, noncuplum, & cyphra numeri 452 addita decussatim, & minuta tribus dextimis notis producunt 267 ferè pro latere icosaedri quæsito. Cæterum utriusque cubatrix communis (quæ est 352) per 1 Problema acquiritur.

Probl. IV.

Duorum corporum regularium eiusdem lateris data cubatrice primi, cubatricem etiam secundi, & utriusque commune latus acquirere.

Exemplum.

Sint duo corpora regularia eiusdem lateris octaedrum & icosaedrum: octaedri cubatrix detur 352, icosaedri autem quæritur. Per 4 theorema ut millenarius se habet ad 352 cubatricem octaedri datam: ita 1694 (numerus interceptus à linea octaedri & columna icosaedri) ad cubatricem icosaedri quæsitam. Vnde triplum, quintuplum, & duplum numeri 1694 (vel contrà) decussatim addita, & minuta tribus dextimis figuris producunt 596 pro cubatrice icosaedri quæsita. Vtriusque prçterea latus commune per 2 Probl. reperitur 452, ut supra.

CAPVT

CAPVT VI.

De inventione cubatricum & diametrorum regularium corporum, & sphæræ per quintam TABVLAM.

Continet hæc Tabula regularium corporum cubatrices & diametros, quas quia sphæra etiam habent, sphæram igitur inter huius tabulæ corpora regularia numeramus. Per corpora itaque regularia hic intellige etiam sphæram, & per diametros corporum regularium intellige sphæræ diametrum, & reliquorum corporum regularium diametrum, majorem scilicet (omissis aliis diametris minùs utilibus) diametrum sphæræ regulari corpori circumscriptæ.

Omnis itaque numerus huius tabulæ vel pro cubatrice, vel pro diametro alicuius corporis regularis accipi potest. Si pro cubatrice, dicetur cubatrix corporis regularis eiusdem lineæ: si pro diametro, dicetur diameter corporis regularis ejusdem columnæ.

Numeri ejusdem columnæ sunt cubatrices corporum regularium ejusdem diametri: & hæc diameter est numerus infimus eiusdem columnæ.

Nu-

Numeri eiusdem lineæ sunt diametri corporum eiusdem cubatricis: & hæc cubatrix est numerus dextimus eiusdem lineæ.

Præcipua Analoga quintæ TABULÆ.

Theor. 1.

Vt millenarius ad cubatricem datam nominati corporis regularis: ita numerus infimus columnæ illius corporis ad diametrum eiusdem corporis.

Theor. 2.

Vt millenarius ad diametrum datam nominati corporis regularis: ita numerus dextimus lineæ illius corporis ad cubatricem eiusdem corporis.

Theor. 3.

Duorum corporum regularium eiusdem diametri ut millenarius ad cubatricem primi datam: ita numerus interceptus à columna primi & linea secũdi ad cubatricem secundi.

Theor. 4.

Duorum corporum regularium eiusdem cubatricis ut millenarius ad diametrum primi datam: ita numerus interceptus à linea primi & columna secundi ad diametrum secundi.

D Tabella

Tabella quinta cubatricum quinque regularium corporum, & diametrorum sphærarum iis circumscriptarum.

Tetrae. 1000	727	693	577	560	496	400
1376	Octae. 1000	953	794	769	683	550
1443	1049	Cubus 1000	833	807	716	577
1732	1260	1201	Icosae. 1000	970	860	693
1487	1300	1238	1031	dodec. 1000	887	715
2015	1465	1396	1163	1127	Sphær. 1000	806
2499	1817	1732	1443	1399	1241	1000

Problemata usus quintæ TABULÆ.

Probl. I.

D*Atâ cubatrice corporis regularis nominati, eiusdem corporis diametrum dare.*

Exem-

Exemplum.

SIT Octaedri cubatrix data 352, ejusdem quæritur diameter. Per 1 theorema ut se habet millenarius ad 352 cubatricem Octaedri datam: ita 1817 (numerus infimus columnę Octaedri) ad diametrum ejusdem quæsitam. Vnde summa ex triplo, quintuplo, & duplo numeri 1817 (vel contrà respectivè) additis decussatim, minuta tribus dextimis figuris, quæ est 639, est diameter petita octaedri, cujus cubatrix dabatur 352.

Probl. II.

Datâ diametro corporis regularis nominati, eiusdem corporis cubatricem invenire.

Exemplum.

Sit Octaedri diameter 639 data, ejusdem cubatrix quæritur. Per 2 theorema ut se habet millenarius ad 639 diametrum Octaedri datam, ita se habebit 550 (numerus dextimus lineæ Octaedri) ad ejusdem Octaedri cubatricem quęsitam. Vnde numeri 629 quintuplum, quintuplum & cyphra (vel contrà numeri 550 sextuplum, triplum, & noncuplum) decussatim addita, & minuta tribus dextimis notis producunt 352 ferè, cubatricem Octaedri quæsitam, cuius scilicet diameter dabatur 639.

PROBL. III.

Duorum corporum regularium eiusdem

D 2 dia-

Table five gives the volumes (or the sides of a cube with an equal volume) and the diameter of containing spheres for regular solids.

diametri, datâ cubatrice primi, cubatricem etiam secundi & utriusque diametrum communem invenire.

Exemplum.

Sint duo corpora ejusdem diametri, Octaedrum primum, & Icosaedrum secundum: Octaedri cubatrix detur 352, Icosaedri quęritur. Per 3 theor. ut se habet millenarius ad 352 cubatricem Octaedri datam: ita 1260 (numerus interceptus à columna octaedri & linea icosaedri) ad cubatricem icosaedri quæsitam. Vnde triplum, quintuplum, & duplum numeri 1260, vel simplum, duplum, sextuplum, & cyphra numeri 352 addita decussatim, & minuta tribus dextimis notis producunt 444 ferè pro cubatrice icosaedri quæsita. Cæterum utriusque diameter communis, quę est 639, per 1 problema acquiritur.

Probl. 4.

Duorum corporum regularium eiusdem cubatricis, datâ diametro primi, diametrum etiam secundi, & utriusque communem cubatricem acquirere.

EXEMPLVM.

Sint duo corpora regularia ejusdem cubatricis octaedrum & icosaedrum: octaedri diameter detur 639, icosaedri autem quæritur. Per 4 theorema ut millenarius se habet ad 639 diametrum octaedri datam: ita 794 (numerus interceptus à linea octaedri

octaedri & columna icosaedri) ad diametrum icosaedri quæsitam. Vnde sextuplum, triplum, & noncuplum numeri 794 (vel contrà) decussatim addita, & minuta tribus dextimis figuris producunt 507, diametrum icosaedri quæsitam. Vtriusque præterea cubatricem communem per 2 problema invenies 352, ut suprà.

CAPVT VII.

De diametris & lateribus quinque corporum regularium per sextam Tabulam inveniendis.

Continet hæc Tabula sexta regularium corporum diametros maiores & latęra, corumque ad invicem proportiones.

Omnis itaque numerus huius tabulæ vel vel pro diametro, vel pro latere alicuius regularis corporis accipi potest. Si pro diametro, dicetur diameter corporis eiusdem lineæ: si pro latere, dicetur latus corporis regularis eiusdem columnæ.

Numeri ejusdem columnæ sunt diametri corporum regularium eiusdem lateris: & hoc latus est numerus supremus eiusdem columnæ.

Numeri eiuſdem lineæ ſunt latera regularium corporum eiuſdem diametri: & hæc diameter eſt primus numerus eiuſdem lineæ.

Præcipua analoga 6 Tabulæ.

THEOR. I.

VT millenarius ad diametrum nominati corporis datam: ita numerus ſupremus columnæ illius corporis, ad corporis ejuſdem latus.

THEOR. II.

Vt millenarius ad datum latus nominati corporis regularis: ita numerus primus lineæ illius corporis, ad corporis eiuſdem diametrum.

THEOR. III.

Duorum corporum regularium eiuſdem lateris, ut millenarius ad diametrum primi: ita numerus interceptus à columna primi & linea ſecundi ad diametrum ſecundi.

THEOR. IV.

Duorum corporum regularium eiuſdem diametri ut millenarius ad latus primi: ita numerus interceptus à linea primi & columna ſecundi ad latus ſecundi.

Tabella

Tabella ſexta laterum quinque regularium corporum, & diametrorũ ſphærarum iis circumſcriptarum.

1000	817	707	577	526	357
1225	*Tetrae.* 1000	966	707	643	437
1414	1035	*Octaed.* 1000	817	742	505
1732	1414	1225	*Cubus.* 1000	909	618
1902	1555	1347	1099	*Icoſaed.* 1000	679
2802	2287	1981	1618	1473	*dodeca.* 1000

Problemata uſus ſextæ TABULAE.

PROBL. I.

DAtâ diametro corporis regularis nominati, eiuſdem corporis latus dare.

EXEMPLVM.

Sit Octaedri diameter 639 data, quæritur

Table six gives the lengths of the sides of regular solids and the diameters of their containing spheres.

tur autem eiuſdem latus. Per 1 theor. ut ſe habet millenarius ad 639 diametrum octaedri datam: ita 707 (numerus ſupremus columnæ octaedri) ad latus octaedri quęſitam. Vnde ſextuplum, triplum, & noncuplum numeri 707: vel ſeptuplum, cyphra, & ſeptuplum numeri 639 addita decuſſatim, & minuta tribus dextimis figuris producunt 452 ferè, pro latere octaedri cuius diameter dabatur 639.

Probl. 2.

Dato latere regularis corporis nominati, eiuſdem corporis diametrum invenire.

Exemplnm.

Sit octaedri latus datum 452, eiuſdem autem diameter quæratur. Per 2 theor. ut ſe habet millenarius ad 452 latus octaedri datum, ita ſe habebit 1414 (numerus primus lineæ octaedri) ad eiuſdem octaedri diametrum quæſitam. Vnde quadruplum, quintuplum, & duplum numeri 1414 (vel contrà) decuſſatim addita, & minuta tribus dextimis figuris producunt 639 pro diametro octaedri quęſita, cuius latus dabatur 452.

Prob. 1.

Duorum corporum regularium eiuſdem lateris datâ diametro primi, diametrum etiam ſecundi & utriuſque latus commune acquirere.

Exem-

Exemplum.

Sint duo corpora regularia, primum octaedrum, ſecundum icoſaedrum eiuſdem lateris: octaedri diameter detur 639, icoſaedri quæratur. Per 3 theorema ut ſe habet millenarius ad 639 octaedri diametrum datam: ita 1347 (numerus interceptus à columna octaedri & linea icoſaedri) ad diametrum icoſaedri quęſitam. Adde ergo decuſſatim ſextuplum, triplum, & noncuplum numeri 1347 (vel contrà) & à producto abſtrahe tres dextimas figuras, & provenient inde 861 ferè, pro diametro icoſaedri quæſita.

Si præterea commune utriuſque latus invenire deſideras, illud per 1 probl. deprehendes eſſe 452.

Probl. IV.

Duorum corporum regularium eiuſdem diametri dato latere primi, latus etiam ſecundi, & utriuſque communem diametrum acquirere.

Exemplum.

Sint duo corpora regularia, primum octaedrum, & ſecundum icoſaedrum eiuſdem diametri: octaedri latus detur 452, icoſaedri quęratur latus. Per 4 theor. ut ſe habet millenarius ad 452 octaedri latus datum: ita ſe habebit 742 (numerus interceptus à linea octaedri, & columna icoſaedri) ad latus Icoſaedri quæſitum. Adde ergo decuſſatim quadruplum,

 quin-

quintuplum, & duplum numeri 742, vel contrà septuplum, quadruplum, & duplum numeri 452, & à producto 335384 abstrahe tres ultimas figuras, & restabunt 335 pro latere Icosaedri quæsito. Vtriusque communem diametrum 639 per 2 problema invenies.

CAPVT VIII.

De ponderibus, & magnitudinibus Metallorum inveniendis.

HActenus præcipuas tum planorum, tum corporum dimensiones expedite & facili compendio invenire docuimus. Libet nunc de Metallorum & lapidum ponderibus & magnitudinibus, corúmque ad invicem proportionibus (quarum apud Mechanichos frequens est usus) hac Tabella septima differere.

Tabellæ septimæ descriptio.

SEptima Tabella Metallorum & Lapidum nomina cum suis millenariis à capite ad calcem decussatim descendentia, complectitur: eorundem etiam Metallarum & Lapi-

Lapidum pondera sub numero drachmarum, & magnitudines sub numero cochlearium continet. Drachma *omnibus est octava pars unciæ.* Cochleare *hîc à nobis usurpatum est pro mensura liquidi, quod à decem auri drachmis in vas liquore plenum injectis expellitur. Unde pro diversitate provinciarum variatâ drachmâ, variatur & etiam cochleare: numeri tamen drachmarum & cochlearium qui in Tabula exprimuntur, eorumque ad inuicem rationes semper invariabiles manent.*

Omnis itaque numerus hujus Tabulæ vel pro drachmis ponderis, vel pro cochlearibus magnitudinis seu capacitatis alicujus metalli & lapidis accipi potest. Si pro drachmis, significat drachmas metalli vel lapidis eiusdem columnæ. Si pro cochlearibus, significat cochlearia metalli, aut lapidis eiusdem lineę.

Numeri eiusdem columnæ sunt cochlearia metallorum vel lapidum eiusdem ponderis: & drachmæ huius ponderis sunt numerus infimus eiusdem columnę.

Numeri eiusdem lineæ sunt drachmæ ponderis metallorum & lapidum eiusdem magnitudinis: & cochlearia hujus magnitudinis sunt numerus dextimus eiusdem lineæ.

Præcipua

Præcipua Analoga septimæ Tabulæ.

THEOR. I.

VT millenarius ad cochlearia capacitatis metalli aut lapidis nominati: ita numerus infimus columnæ illius metalli ad drachmas ponderis eiusdem.

Theor. 2.

Vt millenarius ad drachmas ponderis metalli aut lapidis nominati: ita numerus dextimus lineæ illius metalli ad cochlearia capacitatis eiusdem.

Theor. 3.

Duorum metallorum aut lapidum ejusdem ponderis ut millenarius ad cochlearia capacitatis primi: ita numerus interceptus à columna primi & linea secundi ad cochlearia capacitatis secundi.

Theor. 4.

Duorum metallorum aut lapidum eiusdem magnitudinis ut millenarius ad drachmas ponderis primi: ita numerus interceptus à linea primi & columna secundi ad drachmas ponderis secundi.

PROBLEMATA usus 7 Tabulæ.

Prob. 1.

DAtis cochlearibus capacitatis metalli nominati aut lapidis, drachmas ponderis eiusdem dare.

Exemplum.

Statuæ argenteæ proplasma metitur capacitate 562 cochlearium: quæritur quot drachmas pendat statua? Erit per 1 theorema ut millenarius ad 562 cochlearia capacitatis data: ita 5990 (numerus infimus columnæ argenti) ad drachmas ponderis eiusdem quæsitas. Vnde quintuplum, sextuplum, & duplum numeri 5990 (vel contrà, &c.) Addita decussatim, & minuta tribus dextimis figuris producunt 3366 pro drachmis ponderis statuæ quæsitis, cuius capacitas dabatur 562 cochlearium.

Probl. 2.

Datis drachmis ponderis metalli aut lapidis, cochlearia capacitatis eiusdem acquirere.

Exemplum.

Oblata est statua argentea pendens 3366 drachmas, quæritur quot cochlearium magnitudinem habeat? Per secundum theorema erit ut millenarius ad 3366 drachmas statuæ datas: ita 167 (numerus dextimus lineæ argenti) ad cochlearia capacitatis quæsita.

Vnde simplum, sextuplum, & septuplum numeri 3366 (vel contrà, &c.) addita decussatim, & minuta tribus dextimis notis, producunt 562 pro numero cochlearium capacitatis statuæ quæsito, cuius pondus dabatur 3366 drachmarum.

Prob.

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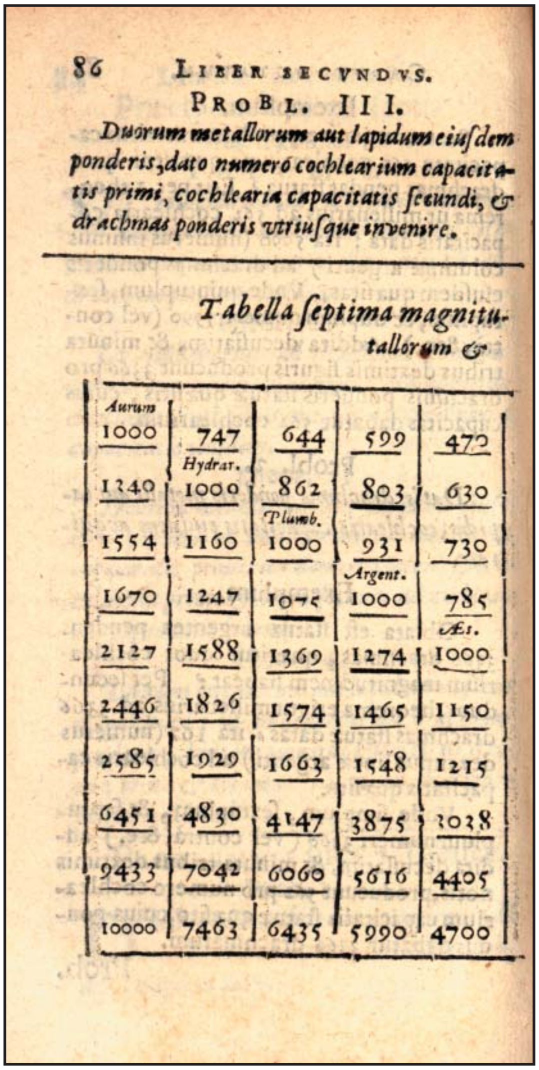

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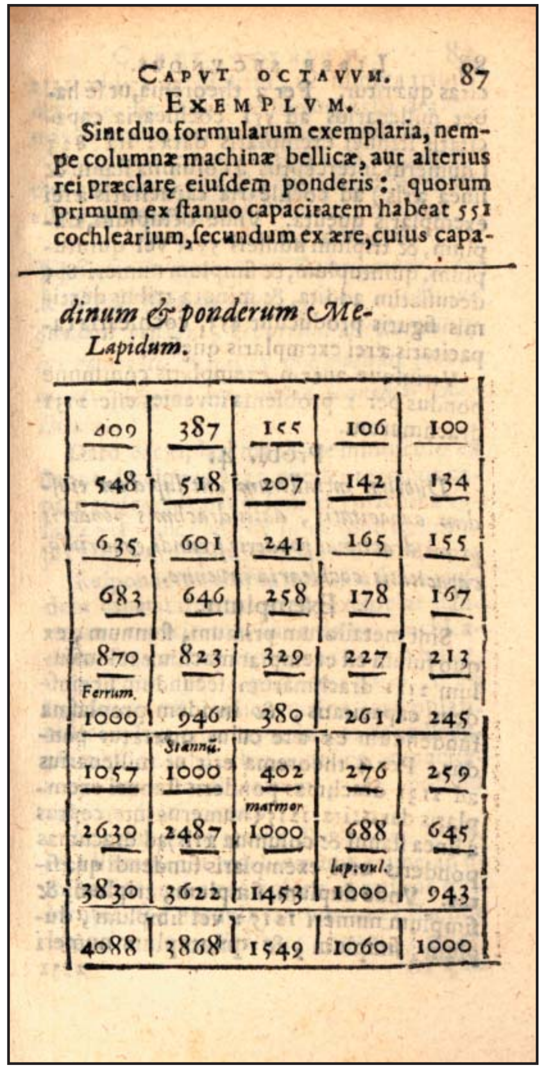
PROBL. III.

Duorum metallorum aut lapidum eiusdem ponderis, dato numero cochlearium capacitatis primi, cochlearia capacitatis secundi, & drachmas ponderis vtriusque invenire.

Tabella septima magnitu- tallorum &

Aurum 1000	747	644	599	472
1340	*Hydrar.* 1000	862	803	630
1554	1160	*Plumb.* 1000	931	730
1670	1247	1075	*Argent.* 1000	785
2127	1588	1369	1274	*Æs.* 1000
2446	1826	1574	1465	1150
2585	1929	1663	1548	1215
6451	4830	4147	3875	3038
9433	7042	6060	5616	4405
10000	7463	6435	5990	4700



CAPVT OCTAVVM. 87

EXEMPLVM.

Sint duo formularum exemplaria, nempe columnæ machinæ bellicæ, aut alterius rei præclarę eiusdem ponderis: quorum primum ex stanuo capacitatem habeat 551 cochlearium, secundum ex ære, cuius capa-

dinum & ponderum Me- Lapidum.

409	387	155	106	100
548	518	207	142	134
635	601	241	165	155
683	646	258	178	167
870	823	329	227	213
Ferrum. 1000	946	380	261	245
1057	*Stannū.* 1000	402	276	259
2630	2487	*marmor.* 1000	688	645
3830	3622	1453	*lap.vul.* 1000	943
4088	3868	1549	1060	1000

Table seven deals with physical properties of metals and stones (gold, mercury, lead, silver, bronze, iron, tin, marble and stone). Of course some of these terms are rather general, but they were simply meant to be used as data for his problems and commercial users would certainly have had their own much more detailed lists to consult.

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citas quæritur. Per 3 theorema, ut se habet millenarius ad 551 cochlearia capacitatis stannei exemplaris data: ita 823 (numerus interceptus à columna stanni & linea æris) ad cochlearia capacitatis ærei exemplaris quęsita. Vnde octuplum, duplum, & triplum numeri 551: vel quintuplum, quintuplum, & simplum numeri 823 decussatim addita, & minuta tribus dextimis figutis producunt 453, cochlearia capacitatis ærei exemplaris quęsita.

Vtriusque autem exemplaris commune pondus per 1 problema invenies esse 2131 drachmarum.

Probl. 4.

Duorum metallorum aut lapidum eiusdem capacitatis, datis drachmis ponderis primi, drachmas ponderis secundi, & utriusq. capacitatis cochlearia invenire.

Exemplum.

Sint metallorum primum, stannum, ex quo fusum est exemplar machinæ minusculum 2131 drachmarum; secundum sit eiusdem capacitatis, & in idem proplasma fundendum ex ære cujus quæratur pondus. Per 4 theorema erit ut millenarius ad 2131 drachmas ponderis stannei exemplaris datas: ita 1215 (numerus interceptus à linea stanni & columna æris) ad drachmas ponderis ærei exemplaris fundendi quæsitas. Vnde duplum, simplum, triplum, & simplum numeri 1215: vel simplum, duplum, simplum, & quintuplum numeri 2131

2131 addita decussatim, & minuta tribus dextimis figuris, producunt 2589 drachmas, pondus ærei exemplaris quæsitum.

Vtriusque autem exemplaris capacitatem communem per 2 problema invenies esse 551 cochlearium.

ADMONITIO.

Præter hos simplices Theorematum, & Problematum usus, qui ex æqualitate quadam pendent, occurrunt alii plurimi ex his compositi, & qui ex inæqualitate proveniunt. Qualis est solutio sequentis quæstionis.

Dato exemplari machinæ minusculo ex stanno drachmas 2131 pendente, cujus capacitati (cochlearium scilicet) machina ipsa ex ære fundenda sit in ratione millecupla: quæritur futuræ machinæ pondus.

Respondetur, si ærea machina foret eiusdem capacitatis cuius est exemplar stanneum, capacitatem haberet 551 cochlearium, & penderet tantum 2589 drachmas, ut per præcedens 4 problema patet. At ex hypothesi est millies major exemplari. Millecuplam ergo capacitatem & millecuplum pondus habebit, videlicet capacitatem 551000 cochlearium, & pondus 2589000 drachmarum.

Longitudines tamen, & diametri, & cætera lineamenta machinæ non erunt ad similia lineaméta exemplaris in ratione millecupla, sed decupla tantum, ut ex Euclide lib.5, definit.10. & lib.11. propos.33. patet.

At quia

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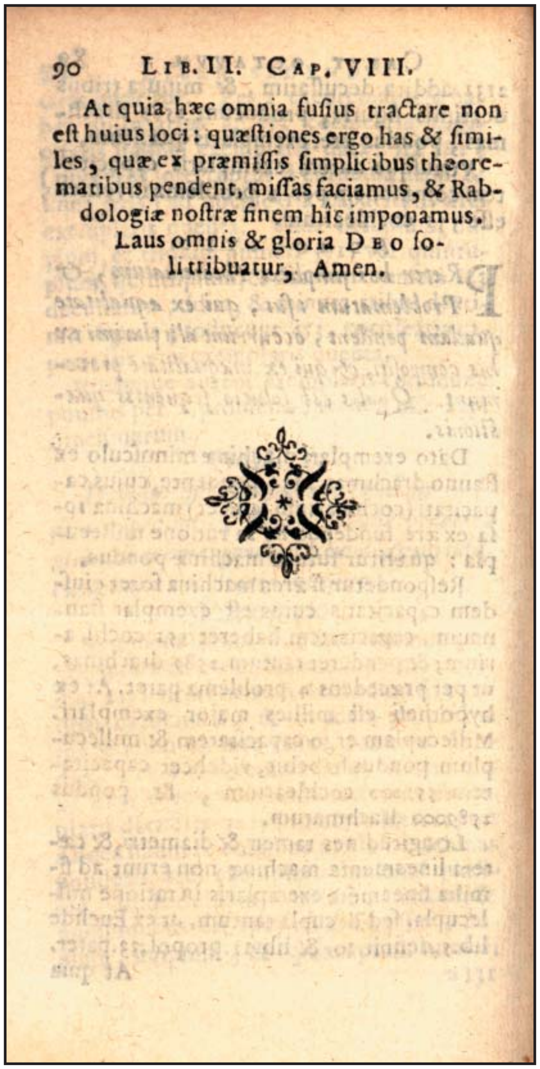
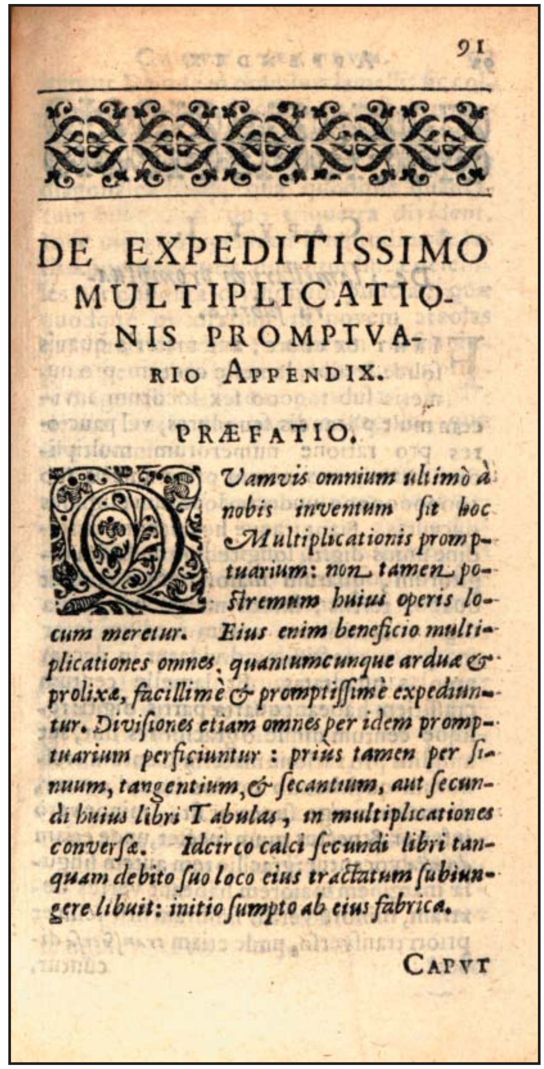

90 Lib. II. Cap. VIII.

At quia hæc omnia fusius tractare non est huius loci: quæstiones ergo has & similes, quæ ex præmissis simplicibus theorematibus pendent, missas faciamus, & Rabdologiæ nostræ finem hîc imponamus.

Laus omnis & gloria Deo soli tribuatur, Amen.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DE EXPEDITISSIMO MULTIPLICATIONIS PROMPTUARIO APPENDIX.

PRÆFATIO.

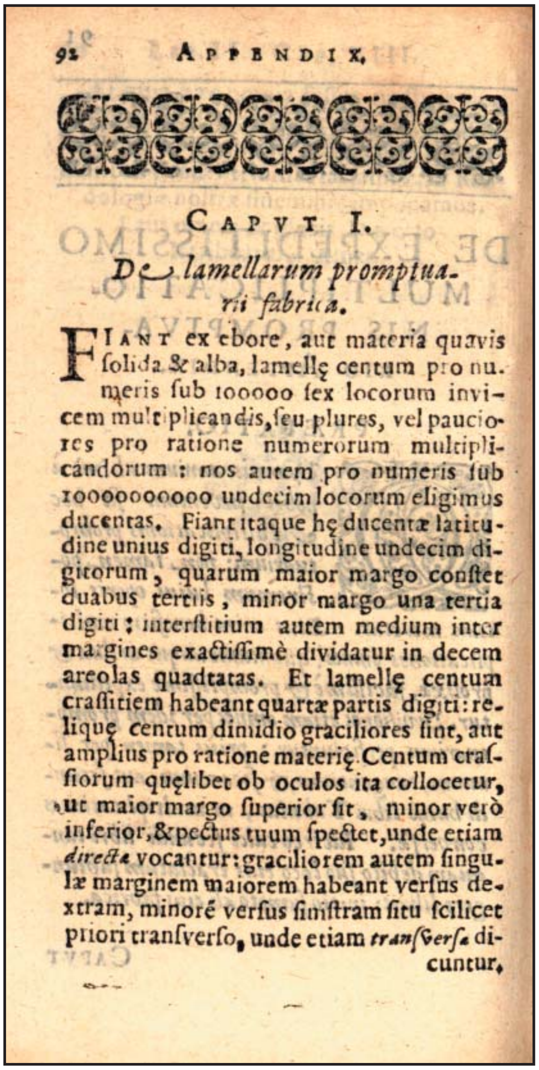
Quamvis omnium ultimò à nobis inventum sit hoc Multiplicationis promptuarium: non tamen postremum huius operis locum meretur. Eius enim beneficio multiplicationes omnes, quantumcunque arduæ & prolixæ, facillimè & promptissimè expediuntur. Divisiones etiam omnes per idem promptuarium perficiuntur: prius tamen per sinuum, tangentium, & secantium, aut secundi huius libri Tabulas, in multiplicationes conversæ. Idcirco calci secundi libri tanquam debito suo loco eius tractatum subiungere libuit: initio sumpto ab eius fabrica.

Caput

The Promptuary of Multiplication

In the preface to his *Promptuary for Multiplication* Napier indicates that this invention was his latest contribution to devices for multiplication and division. Because of its relation to the rods, he thought it best to put it immediately after that material rather than leaving it to the end of the book.

This device was quite complex to make and thus seems to have been little used. Only one early example seems to be preserved in a Madrid museum. For more information on this specimen and a much more detailed description of the device, see "The Promptuary Papers," *Annals of the History of Computing*, Vol. 10, Num. 1, January 1988, pp 35—67.

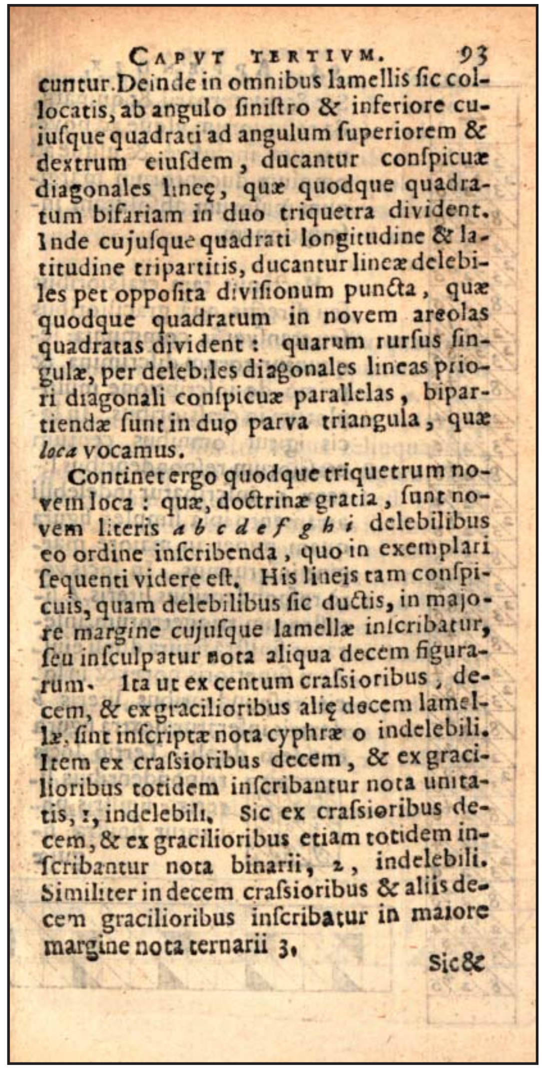


92 APPENDIX.

CAPVT I.

De lamellarum promptuarii fabrica.

FIANT ex ebore, aut materia quavis ſolida & alba, lamellę centum pro numeris ſub 100000 ſex locorum invicem multiplicandis, ſeu plures, vel pauciores pro ratione numerorum multiplicandorum: nos autem pro numeris ſub 10000000000 undecim locorum eligimus ducentas. Fiant itaque hę ducentæ latitudine unius digiti, longitudine undecim digitorum, quarum maior margo conſtet duabus tertiis, minor margo una tertia digiti; interſtitium autem medium inter margines exactiſſimè dividatur in decem areolas quadtatas. Et lamellę centum craſſitiem habeant quartæ partis digiti: reliquę centum dimidio graciliores ſint, aut amplius pro ratione materię. Centum craſſiorum quęlibet ob oculos ita collocetur, ut maior margo ſuperior ſit, minor verò inferior, & pectus tuum ſpectet, unde etiam *directa* vocantur: graciliorem autem ſingulæ marginem maiorem habeant verſus dextram, minoré verſus ſiniſtram ſitu ſcilicet priori tranſverſo, unde etiam *tranſverſa* dicuntur,



CAPVT TERTIVM. 93

cuntur. Deinde in omnibus lamellis ſic collocatis, ab angulo ſiniſtro & inferiore cuiuſque quadrati ad angulum ſuperiorem & dextrum eiuſdem, ducantur conſpicuæ diagonales lineę, quæ quodque quadratum bifariam in duo triquetra divident. Inde cujuſque quadrati longitudine & latitudine tripartitis, ducantur lineæ delebiles per oppoſita diviſionum puncta, quæ quodque quadratum in novem areolas quadratas divident: quarum rurſus ſingulæ, per delebiles diagonales lineas priori diagonali conſpicuæ parallelas, bipartiendæ ſunt in duo parva triangula, quæ *loca* vocamus.

Continet ergo quodque triquetrum novem loca: quæ, doctrinæ gratia, ſunt novem literis *a b c d e f g h i* delebilibus eo ordine inſcribenda, quo in exemplari ſequenti videre eſt. His lineis tam conſpicuis, quam delebilibus ſic ductis, in majore margine cujuſque lamellæ inſcribatur, ſeu inſculpatur nota aliqua decem figurarum. Ita ut ex centum craſsioribus, decem, & ex gracilioribus alię decem lamellæ, ſint inſcriptæ nota cyphræ 0 indelebili. Item ex craſsioribus decem, & ex gracilioribus totidem inſcribantur nota unitatis, 1, indelebili. Sic ex craſsioribus decem, & ex gracilioribus etiam totidem inſcribantur nota binarii, 2, indelebili. Similiter in decem craſsioribus & aliis decem gracilioribus inſcribatur in maiore margine nota ternarii 3.

Sic &

Chapter I: Construction of the device.

The device is composed of two different kinds of strips. These should be made of ivory or some other suitable (he suggests white) material, each about one finger in width and eleven times as long. When dealing with numbers less than 100,000 you should have 100 strips. He is suggesting that 200 would be best as it allows multiplications of numbers less than 10,000,000,000. The strips are easiest to use if half of them are thick (he suggests about a half finger breadth thick) while the other half are much thinner (about half that thickness or less).

The diagram on page 94 shows a sample of both kinds of strips. The middle section of the thick strips (with all the small numbers) is composed of 10 larger squares, each of which is divided into nine little ones. Each of the small squares is divided in half with a diagonal line. The thin strips have similar divisions but contain triangular holes (shown in black in the diagram). He suggests (for a set of 200 strips) that ten of the thicker strips be each noted with the digit “1” another ten with the digit “2” ... to “9”. Similarly, groups of ten of the thin strips should each have the digits “1” to “9” marked in the top section as shown.

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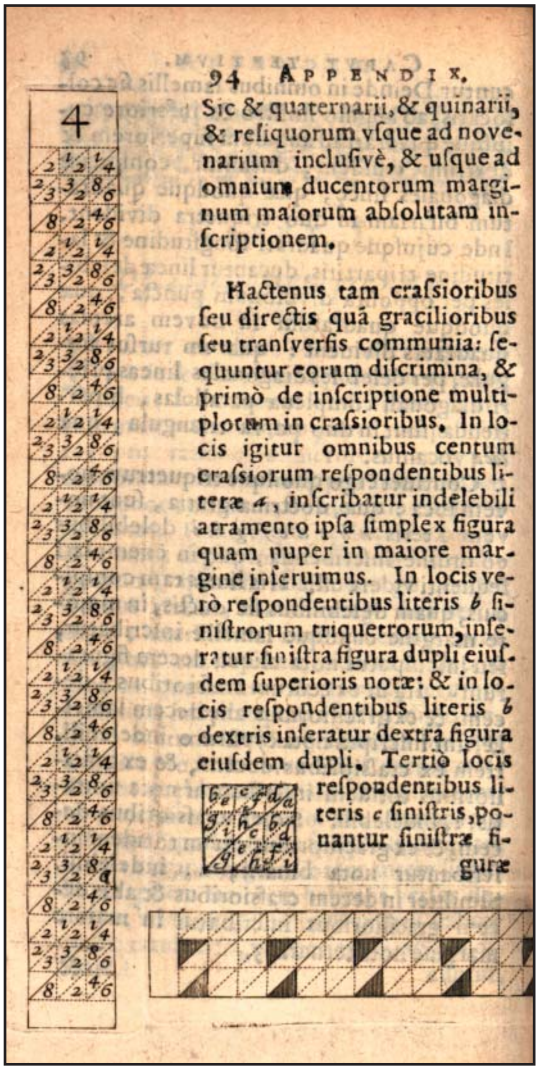
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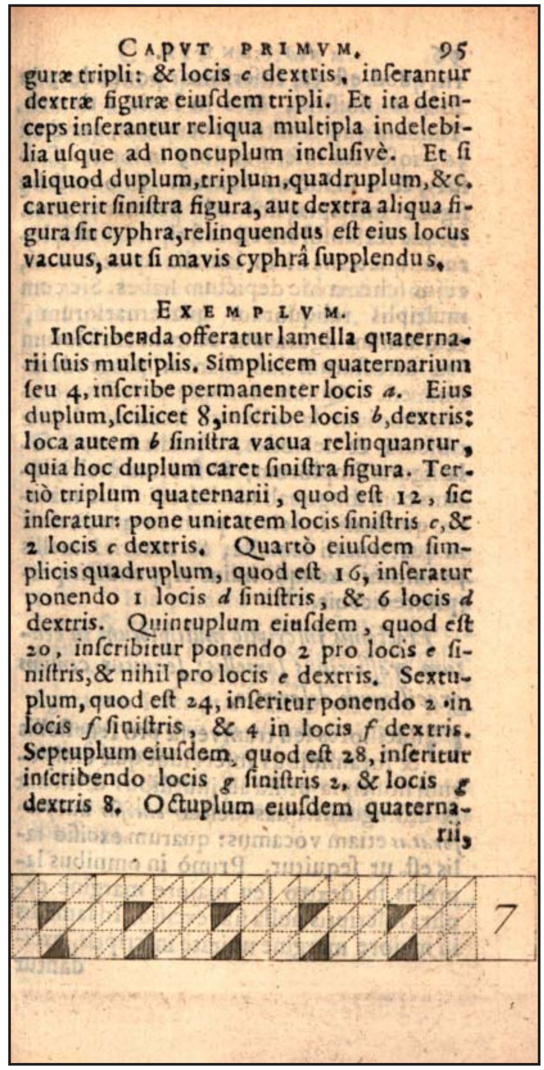
Sic & quaternarii, & quinarii, & reſiquorum vſque ad novenarium incluſivè, & uſque ad omnium ducentorum marginum maiorum abſolutam inſcriptionem.

Hactenus tam craſsioribus ſeu directis quã gracilioribus ſeu tranſverſis communia: ſequuntur eorum diſcrimina, & primò de inſcriptione multiplorum in craſsioribus. In locis igitur omnibus centum craſsiorum reſpondentibus literæ *a*, inſcribatur indelebili atramento ipſa ſimplex figura quam nuper in maiore margine inſeruimus. In locis verò reſpondentibus literis *b* ſiniſtrorum triquetrorum, inſeratur ſiniſtra figura dupli eiuſdem ſuperioris notæ: & in locis reſpondentibus literis *b* dextris inſeratur dextra figura eiuſdem dupli. Tertiò locis reſpondentibus literis *c* ſiniſtris, ponantur ſiniſtræ figuræ


CAPVT PRIMVM. 95

tripli: & locis *c* dextris, inſerantur dextræ figuræ eiuſdem tripli. Et ita deinceps inſerantur reliqua multipla indelebilia uſque ad noncuplum incluſivè. Et ſi aliquod duplum, triplum, quadruplum, &c. caruerit ſiniſtra figura, aut dextra aliqua figura ſit cyphra, relinquendus eſt eius locus vacuus, aut ſi mavis cyphrâ ſupplendus.

EXEMPLVM.

Inſcribenda offeratur lamella quaternarii ſuis multiplis. Simplicem quaternarium ſeu 4, inſcribe permanenter locis *a*. Eius duplum, ſcilicet 8, inſcribe locis *b*, dextris: loca autem *b* ſiniſtra vacua relinquantur, quia hoc duplum caret ſiniſtra figura. Tertiò triplum quaternarii, quod eſt 12, ſic inſeratur: pone unitatem locis ſiniſtris *c*, & 2 locis *c* dextris. Quartò eiuſdem ſimplicis quadruplum, quod eſt 16, inſeratur ponendo 1 locis *d* ſiniſtris, & 6 locis *d* dextris. Quintuplum eiuſdem, quod eſt 20, inſcribitur ponendo 2 pro locis *e* ſiniſtris, & nihil pro locis *e* dextris. Sextuplum, quod eſt 24, inſeritur ponendo 2 in locis *f* ſiniſtris, & 4 in locis *f* dextris. Septuplum eiuſdem, quod eſt 28, inſeritur inſcribendo locis *g* ſiniſtris 2, & locis *g* dextris 8. Octuplum eiuſdem quaternarii,

The small square diagram (annotated with the letters *a*–*g* in the lower triangle and the letters *b*–*g* in the upper triangle) indicates where the various digits are to be placed in the thick strips and the holes in the thin ones.

If you are constructing the thick strip for the digit *x* then write that digit in each place noted by *a*. Take the digits for 2*x* (say *m* and *n*, e.g., for the strip 7, 2*7= 14 so *m* = 1 and *n* = 4) and put *m* in the place noted *b* in the upper triangle and *n* in the place noted by *b* in the lower triangle. Similarly for 3*x* put the *m* and *n* digits in the locations noted with the letter *c*, always putting the tens digit (*m*) in the upper location and the units digit (*n*) in the lower. Continue with this marking until the 9*x* digits are in the locations noted by *i*.

Napier suggests that the lines forming the smallest squares and triangles may be erased leaving only the lines marking the 10 large squares and the diagonal lines of these large squares.

For the thin strips, triangular openings are to be cut in each strip as follows:

Strips for the digit "0" have no openings

Strips for the digit "1" have openings cut in locations noted by *a*

Strips for the digit "2" have openings cut in locations noted by *b*

…

Strips for the digit "9" have openings cut in locations noted by *i*

Once again he suggests that, after the openings are cut, the layout lines may be erased, with the exception of the large squares and their diagonal lines.

rii, quod est 32, inscribitur ponendo 3 in locis *b* sinistris, & 2 in locis *b* dextris. Tandem quaternarii noncuplum, quod est 36, inseritur inscribendo 3 in locis *i* sinistris, & 6 in locis *i* dextris. Et omnes hę figuræ inscriptæ sint ad permanentiam. Atque ita absoluta est inscriptio multiplorum quaternarii in lamella quaternarii, cujus schema hic depictum habes. Sic cum multiplis reliquorum quaternariorum, & omnium figurarum centum crassiorum seu directarum lamellarum progrediendum est. Quibus denique peractis, omnes omnium lamellarum lineæ aut literæ obscuræ & delebiles, delendæ sunt, & solæ figuræ simplorum, & suorum multiplorum cum diagonali media, cuiusque maioris quadrati indeletæ permaneant, veluti in quaternarii lamella, & cæteris lamellis penultimi exempli huius Appendicis perspicere licebit.

Hactenus inscriptio multiplorum in centum crassioribus lamellis: sequitur centum graciliorum descriptio.

GRaciliores seu transversæ pro fenestellis & foraminibus inserviunt quæ crassiorum multipla utilia ab inutilibus dirimant & distinguant: quas idcirco *excisas* aut *perforatas* etiam vocamus: quarum excisio talis est, ut sequitur. Primò in omnibus lamellis in dextro seu majore margine cyphra inscriptis nulla fiat excisio. In lamellis in maiore margine unitate inscriptis, excidantur

dantur loca respondentia literis *a*. In lamellis binario inscriptis, perforentur loca respondentia tam *b* sinistris, quam *b* dextris. In lamellis inscriptis ternario, perforentur omnia loca respondentia utrinq; literis *c*. In lamellis inscriptis quaternario, perforentur loca omnia respondentia literis *d*. In inscriptis quinario, perforentur loca omnia literarum *e*. In inscriptis senario, loca omnia *f* excidantur. In inscriptis septenario, excidantur loca omnia respondentia literis *g*. In octonario inscriptis, perforentur loca omnia literis *h* utrinq; respondentia. Tandem in novenario insculptis lamellis, loca omnia literis *i* tam sinistrorsum quam dextrorsum inscripta excidantur. Et jam habes omnes centum lamellas graciliores debitè perforatas: pro quarum omnium exemplo accipe præcedens schema lamellæ septenarii debitè excisæ. His peractis delendę sunt omnes literæ & lineæ obscurę & delebiles, in areis transversarum inventæ; & solæ diagonales bipartientes quadrata majora, cum notis figurarum inscriptis dextro margini retineantur, veluti in novissimo hujus Appendicis schemate perspicuè apparet.

Atque ita perfecta est omnium ducentarum lamellarum fabrica: sequitur Pyxidis structura.

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CAPVT II.

De Pyxidis, pro continendis lamellis Structura.

AD Pyxidis ſtructuram requiruntur quatuor columnę, duæ tabulæ, & duę regulæ. Columnę ſunt quadratæ, æqualis undique latitudinis, ſcilicet duarum tertiarum digiti: longitudinis verò juxta quinque digitos. Tabulę ſint quadratę, latitudine undique undecim digitorum cum triente: harum altera pro baſi, altera pro ſupremo ſolio ſtatuatur: utraque perforetur quatuor foraminibus quadratis, quorum ſingulorum latitudo ſit tertia pars digiti: & tantum etiam diſtet quodque foramen ab extremis finibus tabularum. Perque hæc foramina ita imponantur quatuor columnæ, ut utrique tabulæ ad rectos angulos directè inſiſtant. Vnde & proxima diſtantia foraminum ab invicem, atque etiam columnarum per ea tranſeuntium, tam ſupra ſolium quam infra, eſt decem digitorum: ut decem lamellarum latitudines tam ſubtus quam ſupra pręciſè capiat: Tabularum autem interſtitium, ſeu columnarùm longitudo inter tabulas, æqualis eſt craſsitiei decem directarum, & totidem tranſverſarum lamellarum: Ita ut hæ viginti lamellæ accumulatæ exactè comprehendantur inter tabu-

tabulas. Duæ tandem regulæ ſint longitudine ęquales latitudini tabularum: earumq. craſsities ſit tertia pars digiti, tanta ſcilicet quantum eſt ſpatium inter foramina & proximas ęxtremitates tabulę: ut ita ſupra margines tabulæ, & ad extremitates columnarum inſtar parietum agglutinari poſſint, altera videlicet ſuper ſiniſtrum marginem, & altera ſuper anteriorem marginem tabulæ. Sitque ſingularum latitudo ſeu altitudo æqualis craſsitiei duarum lamellarum, altera craſsiore, altera graciliore. Denique quicquid columnarum his regulis altius ſupereminet abſcindatur.

Cæterùm Pyxidis partibus hoc ſitu conglutinatis, dividendæ ſunt longitudines exteriorum octo facierum quatuor columnarum inter tabulas interjectæ, in decem æquales partes: quarum rurſus quælibet dividēda eſt in duas inæquales partes, alteram inferiorem, majuſculam, & æqualem craſsitiei lamellę craſsioris: alteram ſuperiorem, minuſculam, & æqualem craſsitiei lamellæ gracilioris. Deinde in infima diviſione majuſcula anteriorum & poſteriorum facierum inferantur figuræ novenarii. Et ſupra hanc aſcendendo ad ſequentem maiuſculam diviſionem quatuor columnarum (omiſsis minuſculis) inſerantur figuræ octonarii. Et in tertiis majuſculis diviſionibus earundem facierum inſcribatur ſeptenarius. Et ita aſcendendo per majuſculas diviſiones anteriorum & poſteriorum facierum uſque ad cyphram incluſivè

Chapter II: Construction of the box holding the strips.

Napier suggests that a box be made (see diagram after page 100) to hold the two different kinds of strips. The numbered strips fitting in one side and the perforated strips in the other. The top of the box should be a flat surface with guides on two edges so that the strips may be placed (thick numbered strips vertically and thin perforated strips horizontally on top of the thick ones) when performing an operation.

sive inserantur reliquæ figuræ senarii, quinarii, quaternarii, &c. Quibus insertis incipe rursus ab infima divisione minuscula facierum dextrarum & sinistrarum (omissis hic omnibus majusculis) in qua inscribatur novenarius. Et supra hanc ascendendo scribe in sequente earundem facierum divisione minuscula figuram octonarii. Et supra hanc in tertia minuscula earundem facierum septenarium: & proinde senarium, quinarium, & cæteras figuras ascendendo usque ad cyphram inclusivè. Et ita absoluta est pyxidis structura, & columnarum ejus inscriptio; secundum quam hoc modo inserendæ sunt lamellę pyxidi.

Pyxide igitur ita statuta, ut altera regula sit versus sinistram, altera versus pectus tuum, decem directæ lamellæ figura novenarii inscriptę supersternantur basi inter figuras anteriores novenarii 9 & 9; ita ut facies inscripta cœlum, non inscripta humum; major margo posteriorem pyxidis faciem, minor anteriorem spectet: lamellæ enim directæ sic insertæ dicuntur *debitè insterni*. Deinde accipe decem ex transversis seu gracilioribus lamellis figura novenarii inscriptis, & has illis ex transverso inter figuras dextras 9 & 9 supersternito; ita ut major margo dextram, minor sinistram, facies inscripta cœlum, non inscripta humum spectent: & lamellæ transversæ sic insertæ dicuntur *debitè insterni*. Secundò accipe decem lamellas directas octonario inscriptas, & has præmissis inter figuras anterio-

anteriores 8 & 8 debitè insternito. Proinde super has, decem ex transversis inscriptis octonario debitè (id est transversim) inter figuras dextras 8 & 8 sternito. Tertiò decem ex directis septenario inscriptæ, debitè super has transversas inter anteriores figuras 7 & 7 insternantur. Et super has rursus decem ex transversis septenario inscriptæ inter figuras dextrarum columnarum 7 & 7 debitè insternantur. Quartò decem ex directis senario inscriptæ debitè super has inter 6 & 6 anteriorum columnarum insternantur. Et his rursus decem transversæ senario inscriptæ inter 6 & 6 dextrarum columnarum debitè insternantur. Et ita insternendo directas lamellas quinarii, quaternarii, ternarii, &c. anteriùs; & transversas quinarii, quaternarii, ternarii, &c. dextrorsum, debitè & inter suas figuras in columnis notatas, alternatis vicibus progredere usque ad cyphras 0, & pyxidis repletionem. Et pyxidem sic repletam *promptuarium* dicimus; cujus fabricam jam absolutam habes, cum ejusdem schemate hic annexo.

Iuxta hunc locum inseritur schema promptuarii notatum literâ. A.

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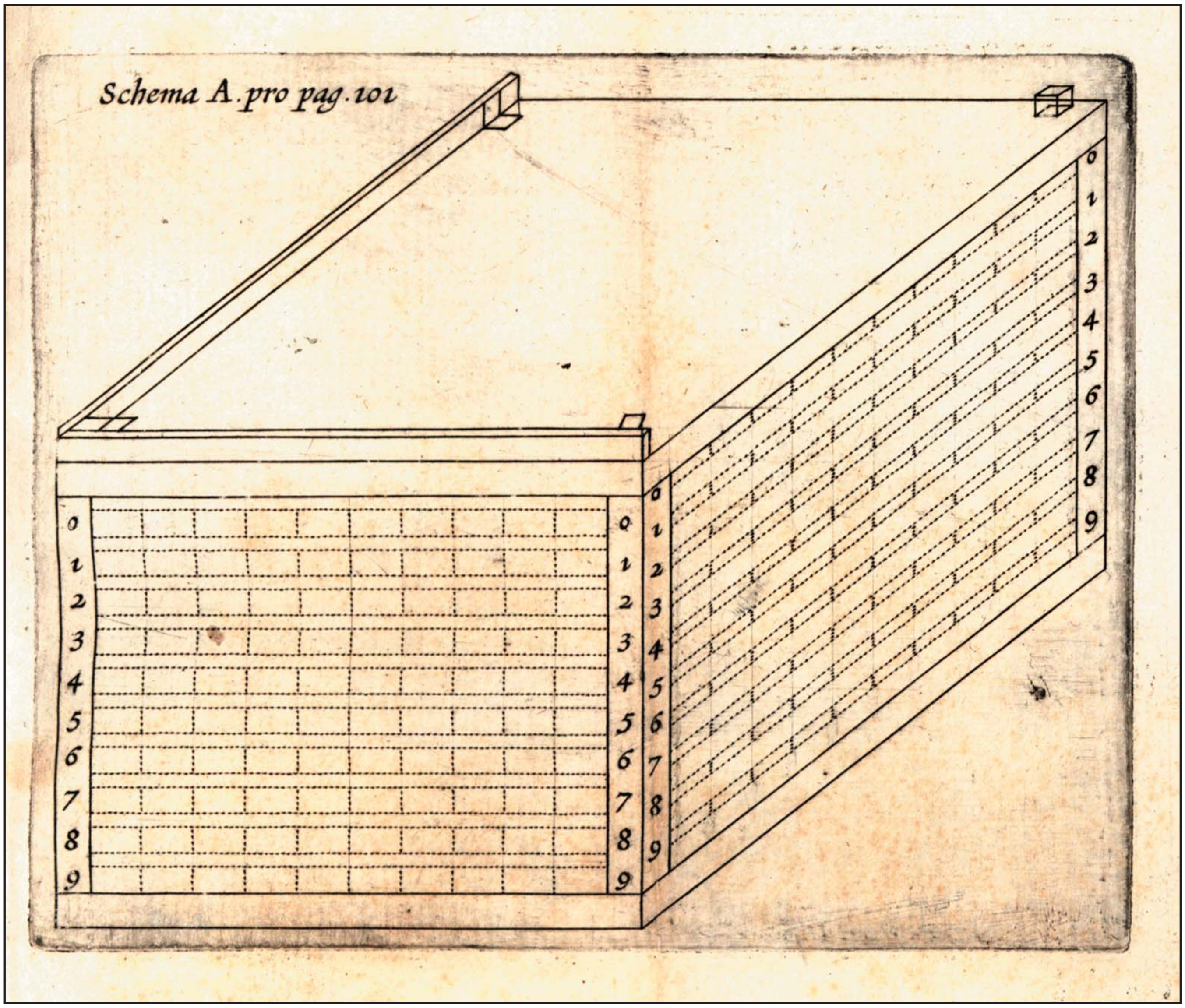

CAPVT III.

De facili per promptuarium Multiplicatione.

PRomptuarii usus potissimum in Multiplicatione spectatur. In multiplicatione autem requiritur debita dispositio multiplicandi & multiplicantis, in supremo Pyxidis folio. Multiplicandi quidem dispositio fit ad hunc modum: Pro prima seu dextima figura multiplicandi pone in primo & dextimo folii loco lamellam directam figura prima multiplicandi inscriptam, è pyxide directè sub loco hoc dextimo folii desumptam. Pro secunda figura multiplicandi, pone secundo folii loco lamellam directam secunda multiplicandi figura inscriptam è pyxide sub hoc secundo folii loco depromptam. Sic pro tertia, quarta, quinta, & cæteris multiplicandi figuris dispone tertio, quarto, quinto, & reliquis locis lamellas directas, tertia, quarta, quinta, & cæteris multiplicandi figuris, inscriptas, è pyxide sub iisdem locis respectivè depromptas usque ad ultimam multiplicandi figuram: repletis locis omnibus sinistris (si quæ vacua sint) lamellis cyphrâ in-

scriptis

scriptis ad arctiorem totius folii repletionem. Et ita habes multiplicandum in folio dispositum.

Superest multiplicantem folio inserere, quod sic fit: Pro prima seu dextima figura multiplicantis, superpone directis ex transverso in primo & anteriore folii loco, lamellam transversam prima multiplicantis figura inscriptam è pyxide directè sub loco hoc folii anteriore desumptam. Pro secunda multiplicantis figura, transversim superpone lamellis directis in secundo loco, lamellam transversam secunda multiplicantis figura inscriptam, è pyxide sub hoc secundo loco depromptam. Sic pro tertia, quarta, quinta, & reliquis multiplicantis figuris: directis ex transverso supersterne in tertio, quarto, quinto, & reliquis locis, lamellas transversas tertia, quarta, quinta, & cæteris multiplicandi figuris inscriptas, è pyxide sub iisdem folii locis respectivè depromptas, usque ad ultimam multiplicantis figuram: repletis & hic locis, tot lamellis cyphra 0 inscriptis, quot fuerint loca vacua.

Atque ita iam habes tam multiplicantem quàm multiplicandum in folio ritè dispositos: & simul cum illis in area dispersas figuras producti ex multiplicatione eo-

Chapter III: Use of the promptuary of multiplication.

The device is used by placing the thick numbered strips corresponding to the multiplicand on top of the box and laying the thin perforated strips representing the multiplier at right angles over the previous ones.

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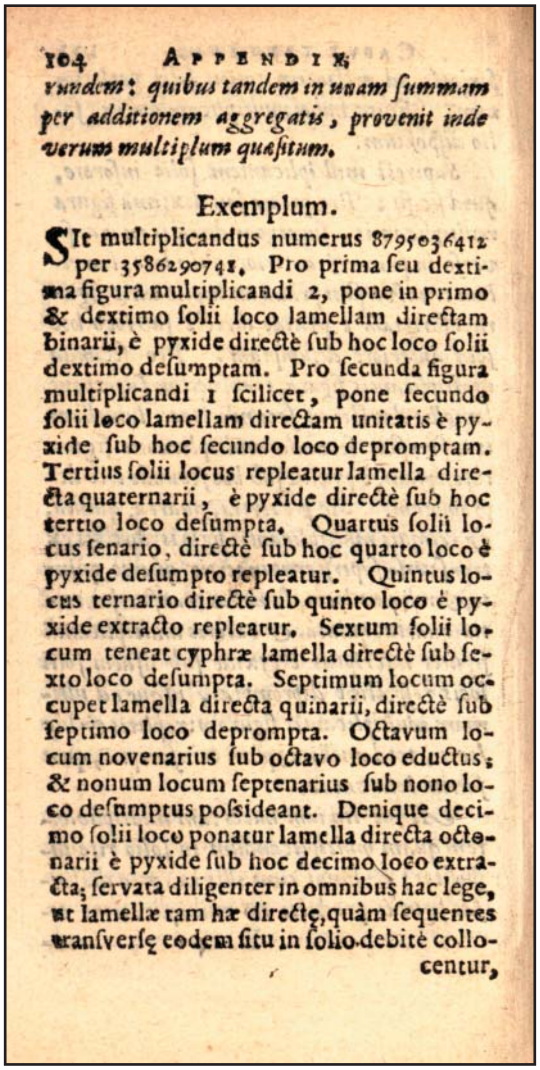
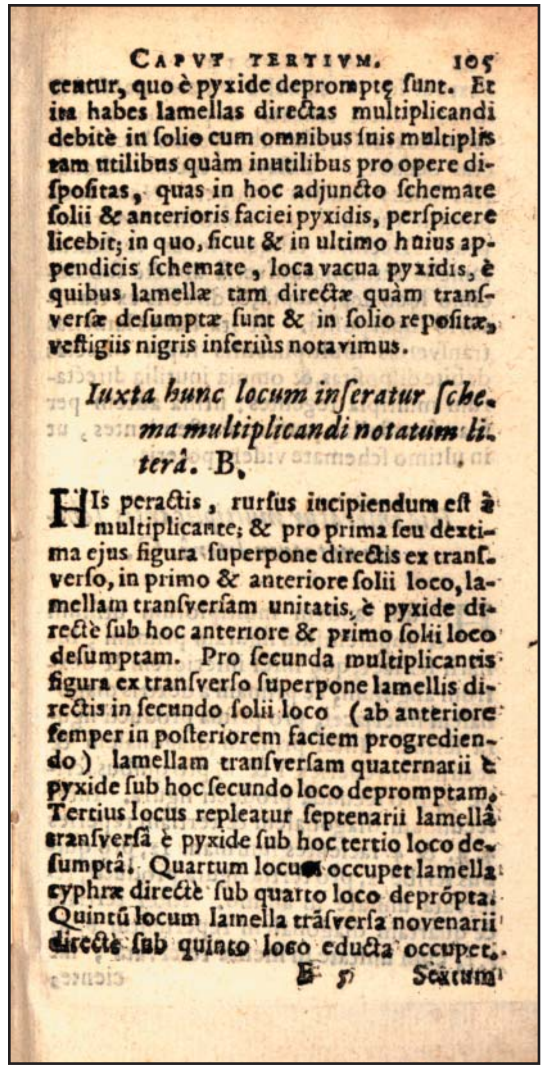


104 APPENDIX.

rundem: quibus tandem in unam ſummam per additionem aggregatis, provenit inde verum multiplum quæſitum.

Exemplum.

Sit multiplicandus numerus 8795036412 per 3586290741. Pro prima ſeu dextima figura multiplicandi 2, pone in primo & dextimo ſolii loco lamellam directam binarii, è pyxide directè ſub hoc loco ſolii dextimo deſumptam. Pro ſecunda figura multiplicandi 1 ſcilicet, pone ſecundo ſolii loco lamellam directam unitatis è pyxide ſub hoc ſecundo loco depromptam. Tertius ſolii locus repleatur lamella directa quaternarii, è pyxide directè ſub hoc tertio loco deſumpta. Quartus ſolii locus ſenario, directè ſub hoc quarto loco è pyxide deſumpto repleatur. Quintus locus ternario directè ſub quinto loco è pyxide extracto repleatur. Sextum ſolii locum teneat cyphræ lamella directè ſub ſexto loco deſumpta. Septimum locum occupet lamella directa quinarii, directè ſub ſeptimo loco deprompta. Octavum locum novenarius ſub octavo loco eductus; & nonum locum ſeptenarius ſub nono loco deſumptus poſſideant. Denique decimo ſolii loco ponatur lamella directa octonarii è pyxide ſub hoc decimo loco extracta; ſervata diligenter in omnibus hac lege, ut lamellæ tam hæ directę, quàm ſequentes tranſverſę eodem ſitu in ſolio debitè collocentur,



CAPVT TERTIVM. 105

centur, quo è pyxide depromptę ſunt. Et ita habes lamellas directas multiplicandi debitè in ſolio cum omnibus ſuis multiplis tam utilibus quàm inutilibus pro opere diſpoſitas, quas in hoc adjuncto ſchemate ſolii & anterioris faciei pyxidis, perſpicere licebit; in quo, ſicut & in ultimo huius appendicis ſchemate, loca vacua pyxidis, è quibus lamellæ tam directæ quàm tranſverſæ deſumptæ ſunt & in ſolio repoſitæ, veſtigiis nigris inferiùs notavimus.

Iuxta hunc locum inſeratur ſchema multiplicandi notatum literà. B.

His peractis, rurſus incipiendum eſt à multiplicante; & pro prima ſeu dextima ejus figura ſuperpone directis ex tranſverſo, in primo & anteriore ſolii loco, lamellam tranſverſam unitatis, è pyxide directè ſub hoc anteriore & primo ſolii loco deſumptam. Pro ſecunda multiplicantis figura ex tranſverſo ſuperpone lamellis directis in ſecundo ſolii loco (ab anteriore ſemper in poſteriorem faciem progrediendo) lamellam tranſverſam quaternarii è pyxide ſub hoc ſecundo loco depromptam. Tertius locus repleatur ſeptenarii lamellâ tranſverſâ è pyxide ſub hoc tertio loco deſumptâ. Quartum locum occupet lamella cyphræ directè ſub quarto loco deprõpta. Quintũ locum lamella trãſverſa novenarii directè ſub quinto loco educta occupet.

E 5 Sextum

He proposes to demonstrate the process with the example of 8,795,036,412 times 3,586,290,741.

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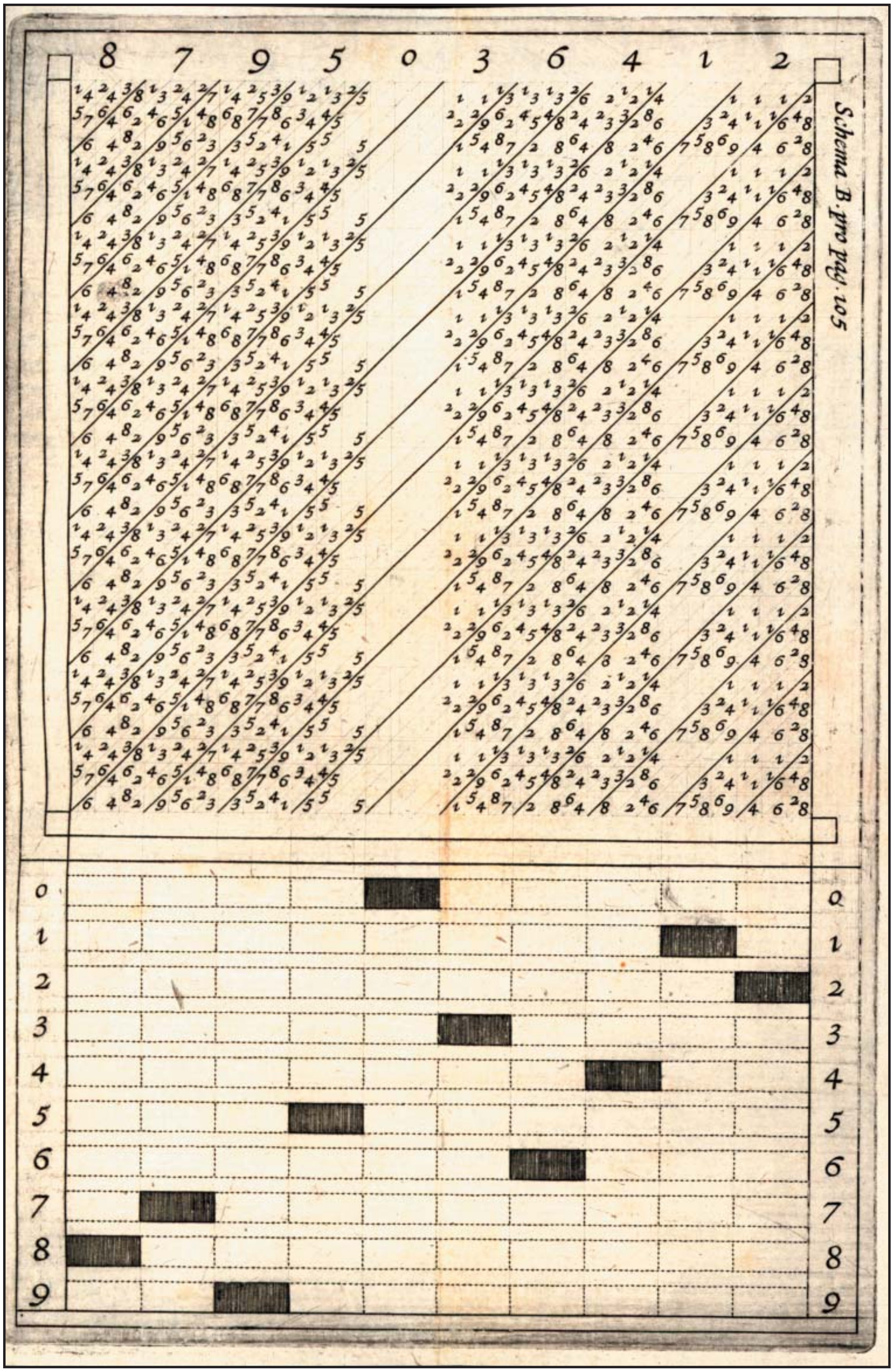


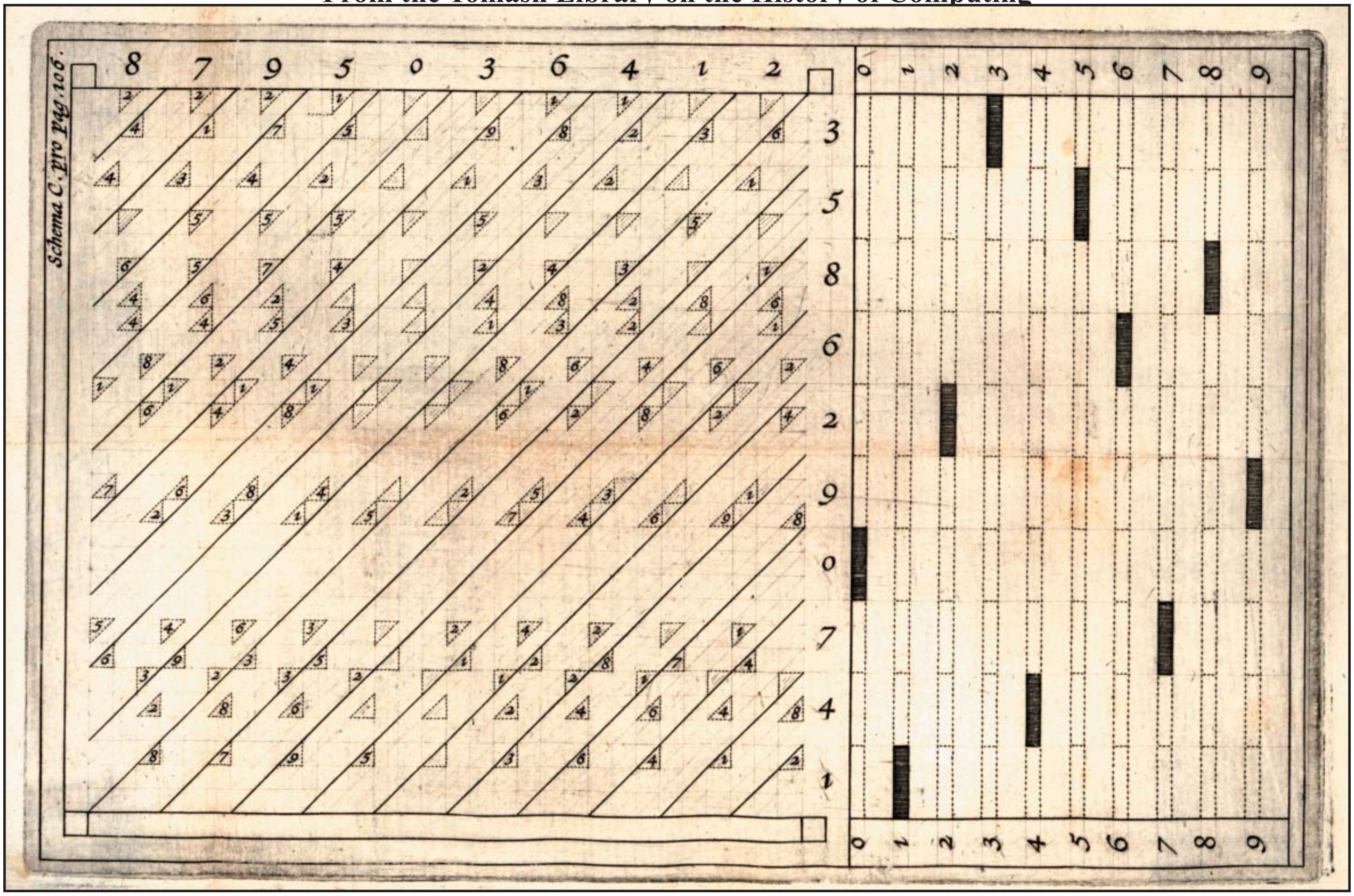
The strips for the multiplicand (8795036412) are placed on top of the box. The lower portion of this diagram is intended to represent the front of the box with the black areas indicating the storage locations from which the individual strips were drawn.

Sextum locum lamella binarii tranſverſa ſubtus educta teneat. Septimum locum ſenarius tranſverſus ſub ſeptimo loco eductus occupet. Octavum locum octonarius ſubtus eductus occupet. Nono loco ponatur quinarius ſub nono loco depromptus. Decimus tandem locus repleatur lamellâ ternarii tranſverſâ directè ſub decimo loco depromptâ, & directis ex tranſverſo ſuperpoſitâ. Et ita habes lamellas tranſverſas multiplicantis ſuper directas debite diſpoſitas, & omnia inutilia directarum multipla tegentes; utilia autem per ſuas feneſtellas perſpicuè oſtendentes, ut in ultimo ſchemate videre poteris.

Hic inſeratur multiplicantis ſchema notatum literâ. C.

HOrum tandem multiplorum utilium & apparentium figuram primam, binarii ſcilicet, quę inter inferiorem & dextrum angulum, ac primam à dextris diagonalem interjacet, pro prima producti figura ſcribe. Inter primam diagonalem & ſecundam reperies 1 & 8, pro quibus ſcribe 9, pro ſecunda producti figura. Inter ſecundam diagonalem & tertiam reperies 4. 4. & 4. facientes ſummam 12, pro quibus ſcribe 2, pro tertia figura producti, reſervatâ unitate in animo. Inter tertiam & quartam diagonalem reperiuntur 6. 6. 7, 1, cum unitate in mente reſervata, fa-

cientes 21, quorum 1 ſcribitur pro quarta figura producti, & 2 in animo reſervantur. In quinto loco ſeu intervallo (ſcilicet inter quartam & quintam diagonales) ſunt 3. 4 1. 8. 8, quæ cum binario mente reſervato producunt 26, quorum 6 ſcribuntur pro quinti loci figura, & 2 animo reſervantur. In ſexto intervallo ſunt 2. 2. 2. 2. 9. 1. 4, cum binario mente ſervato, faciences 24, quorum 4 ſunt figura ſexti loci, & binarius animo reſervatur. Septimo intervallo reperiuntur figuræ, quæ cum præcedente mentis binario efficiunt 23, hoc eſt 3 pro ſeptima producti figura, & 2 in animo. Octavo intervallo reperiuntur cum his in animo 41, ſcilicet unitas ſcribenda octavo loco, & quaternarius ſequentibus annumerandus, qui cum figuris noni intervalli efficiunt 51, hoc eſt 1 nono loco, & 5 in mente. Quæ 5 rurſus cum decimi intervalli figuris efficiunt 61, hoc eſt 1 decimo producti loco, & ſenarium mente reſervandum. Qui cum reliquis undecimi intervalli figuris efficit 55, ſcilicet 5 reponenda undecimo loco, & 5 figuris duodecimi intervalli annumeranda. Quæ quidem 36 efficiunt, 6 ſcilicet duodecimo loco, & 3 decimotertio intervallo annumeranda. Atque hac vulgari ARITHMETICES methodo ſervatâ reperies figuram decimitertii loci eſſe 7, decimiquarti 5, decimiquinti 5, decimiſexti 1, decimiſeptimi 4, decimioctavi 5, deciminoni 1, & denique vigeſimi loci in pro-



The perforated strips for the multiplier (3,586,290,741) are now placed horizontally on top of the vertical strips for the multiplicand. The right hand portion of this diagram represents the right side of the box with the black areas indicating the storage locations from which the individual strips were drawn.

The various digits of the product can now be determined by adding up the numbers visible in the windows in each diagonal line beginning with units digit in the lower right diagonal (2). The tens digit is found by adding up the digits visible in the next higher diagonal (8+1 = 9). The hundreds digit is the sum of the digits in the third diagonal (4+4+4 = 2 and carry the 1 to the next diagonal). The thousands digit is 1+7+6+6 + (the carried 1 from the previous sum) = 1 and carry the 2 to the next diagonal sum.

Performing this summation over all the diagonals, the product is found to be 31,541,557,651,113,461,292.

Care has to be taken with this diagram because the modern reader will, at first glance, easily confuse the “1” and “2” digits shown.

The diagram is shown here rotated 90º from the original—simply because it is easier to read that way.

108 APPENDIX.

producto 3. Atque ita tota ſumma ex multiplicatione producta, & à nobis quæſita, eſt 31541557651113461292. Quâ quidem (memoriæ gratia) in charta notatâ, omnes tandem ſupremi ſolii lamellę tam directæ quàm tranſverſæ ſunt iterum ad ſua priſtina loca in pyxide vacua, & nigris veſtigiis in ſchemate à nobis ſignata, ac directè ſub locis ſolii conſtituta, revocandæ & reſtituendę; ut ita ſemper promptuarium promptum & paratum ad alias atque alias multiplicationes perficiendas maneat. Similiter in aliis exemplis progredere.

CAPUT IV.

De diviſione per promptuarium, & Tabulas.

Diviſio non abſolvitur per promptuarium noſtrum niſi prius converſa in multiplicationem: in hac converſione mutandus eſt diviſor in ſuum extremum relatum.

Extremum relatum eſt numerus ita ſe habens ad medium relatum; ut medium relatum ad primum numerum oblatum.

Medium relatum eſt ſemper unitas cum cyphris aliquot illi verſus dextram appoſitis.

Unde

CAPUT QUARTUM. 109

Unde in quibuſdam authorum tabulis medium relatum eſt 1000 quatuor locorum, ut in ſecundo libro præcedente RABDOLOGIÆ noſtræ. In aliis autem authoribus eſt 100000 ſex locorum, ut in manuali pitiſci ſinuum, tangentium, & ſecantium. In aliis authoribus eſt 10000000 octo locorum, ut in canone ſinuum, tangentium, & ſecantium LANSBERGII. In aliis authoribus aliud eſt, ſemper tamen unitate & cyphris notatum, quod vulgò ſinum totum vocant.

Exemplum.

Ut quibus medium relatum, ſeu ſinus totus eſt 1000, & 125 numerus oblatus, erit 8000 ejus extremum relatum: quia 8000 ita ſe habent ad 1000, ut 1000 ad 125.

Unde medium relatum ſeu ſinus totus eſt ſemper medium proportionale inter quemlibet numerum & ſuum extremum relatum.

Atque etiam factum ex numero aliquo & ſuo extremo relato æquatur quadrato ſinus totius, ſeu (ut nos dicimus) quadrato relati medii.

Ut 8000 ducta in 125, idem producunt, quod medium relatum 1000 in ſe ductum, ſcilicet 1000000.

Hæc extrema relata ſolent in Tabulis ex diametro ſuis numeris datis opponi, aut in locis adeò conſpicuis inſcribi, ut altero invento reliquum extemplò inveniatur.

Unde

Chapter IV: Division using the promptuary and tables.

Napier had earlier remarked that the promptuary was really only for multiplication but felt compelled to add a short section here on how it might be used for division. Essentially one had to convert the division problem into one of multiplication and then solve that for the product. He explains how this might be done by examining numbers in the various tables of trigonometric functions (those by Pitiscus, Lansberge and his own tables earlier in this volume).

The method is difficult and requires knowledge of each set of tables. Users would be advised to heed Napier's original remark and not consider using the promptuary for anything except multiplication.

Vnde in Pitisci canone, dati & sui extremi relati, altero in sinuum columna prima invento, alterum in secantium columna ultima illi è regione invenietur: Aut altero, in secunda quæ tangentium est columna invento, alterum in penultima paginæ columna invenietur è regione. Aut deniq; altero in tertia columna invento, alterum illi è regione invenietur in antepenultima columna.

LANSBERGIUS autem habet datorum & suorum relatorum extremorum alterum inter sinus arcuum, alterum inter secantes complementorum eorundem: vel alterum inter tangentes arcuum, alterum inter tangentes suorum complementorum. Et nos quidem in secundo Libro RAEBDOLOGIÆ hujus ponimus bina extrema relata (quorum alterū datur, alterum quęritur) in eadem diagonali linea æqualiter à media millenariorum linea distantia. Veluti in prima Tabella, 658, & 1520 sunt extrema relata: Item 502, & 1991 sunt etiam extrema relata: similiter 408, & 2450: vel 702, & 1312 sunt extrema relata. Et ita de aliis omnibus huius libri extremis relatis.

Si ergò, his intellectis, offeratur tibi numerus per numerum dividendus, convertes divisionem in multiplicationem hoc modo.

Multiplica dividendum oblatum per

divi-

divisoris dati extremum relatum: producto suppone (fractionum more) quadratum medii relati: aut illi à dextris aufer tot figuras, quot sunt in hoc cyphrę: & proveniet inde optatus quotiens divisionis imperatæ.

EXEMPLUM.

VT ex Tabulis LANSBERGII sit dividendus 8795036412, per 27884. Per præmissam multiplicationis regulam multiplicabis 8795036412, per extremum relatum numeri 27884, quod est 3586290741: & inde producentur 31541557651113461292: & huic producto suppone quadratum medii relati, seu quadratum sinus totius, quod LANSBERGIO est 100,000,000,000,000, quindecim locorum, & fient inde more fractionum $\frac{31541557651113461292}{1000000000000000}$, seu per distinctionem integrorum à fractis sic $315415\frac{57651113461292}{1000000000000000}$: vel per fractionis omissionem sic 315415, pro quotiente divisionis desiderato.

Aliud Exemplum.

Positâ Tabulâ cujus sinus totus seu medium relatum sit 10000000000 undecim locorum: & ex hac Tabula sit dividendus 8795036412, per 27883963465. Per promptuarium nostrum multiplica, 8795036412, per numeri 27883963465

extre-

extremum relatū, quod est 3586290741; & inde (ut superiùs) producentur 31541557651113461292; & huic producto supponatur medii relati quadratum, quod est 100000000000000000000 viginti unius locorum, & proveniet inde hæc fractio $\frac{31541557651113461292}{100000000000000000000}$ pro quotiente divisionis vero quæsito. Et ita progredere in omnibus divisionibus oblatis, atque eas Tabularum ope in multiplicationes converte, & facillimè inde dabit hoc promptuarium optatum quotientem. Itaque absolutis jam fabricâ & usu hujus promptuarii, ad Arithmeticam localem procedamus.

LAUS DEO.

ARITHMETICAE LOCALIS, quæ in SCACCHIÆ abaco exercetur, LIBER UNUS.

PRÆFATIO.

DUM in his calculi compendiis (quoties per otium licuerat) investigandis operam aliquando darem, & quibus modis labor & molestia calculi tolleretur, inquirerem: incidi (præter LOGARITHMOS, RABDOLOGIAM, PROMPTUARIUM *Multiplicationis, & alia) in tabularem quandam Arithmeticam, quæ (quum omnia graviora Arithmetica vulgaris opera in abaco seu area Scacchiæ perficiat) meritò ludus,*

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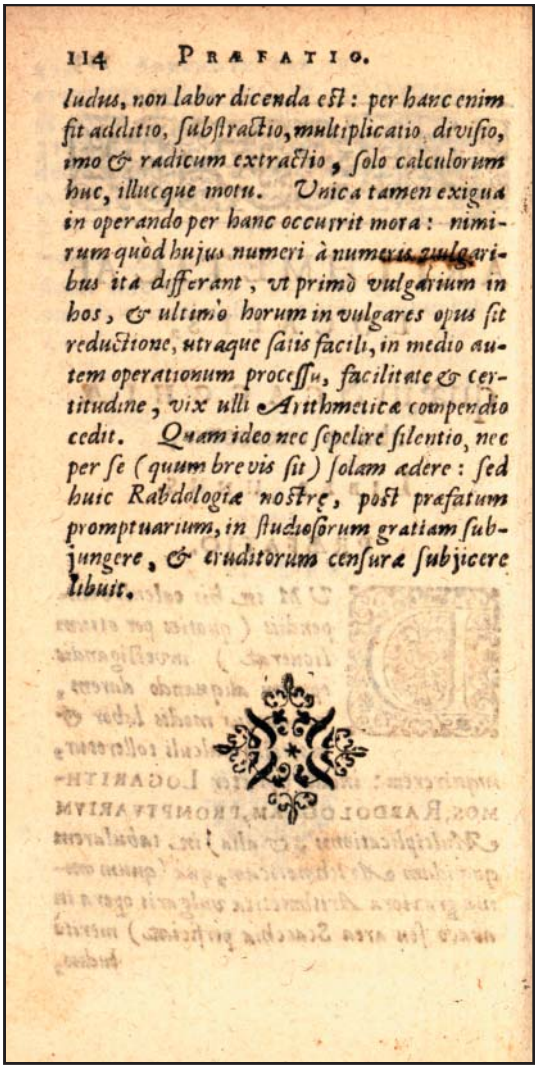
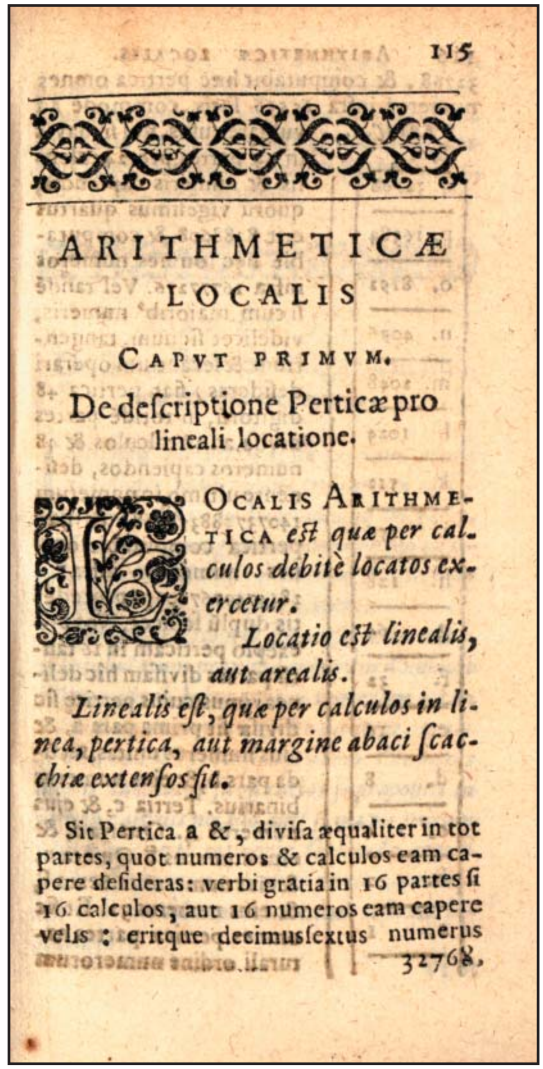

114 PRÆFATIO.

ludus, non labor dicenda est: per hanc enim fit additio, subſtractio, multiplicatio, diviſio, imo & radicum extractio, ſolo calculorum huc, illucque motu. Unica tamen exigua in operando per hanc occurrit mora: nimirum quòd hujus numeri à numeris vulgaribus ita differant, ut primò vulgarium in hos, & ultimò horum in vulgares opus ſit reductione, utraque ſatis facili, in medio autem operationum proceſſu, facilitate & certitudine, vix ulli Arithmeticæ compendio cedit. Quam ideo nec ſepelire ſilentio, nec per ſe (quum brevis ſit) ſolam edere: ſed huic Rabdologiæ noſtræ, poſt præfatum promptuarium, in ſtudioſorum gratiam ſubjungere, & eruditorum cenſuræ ſubjicere libuit.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ARITHMETICÆ LOCALIS

CAPUT PRIMUM.

De deſcriptione Perticæ pro lineali locatione.

LOCALIS ARITHMETICA *eſt quæ per calculos debitè locatos exercetur.*

Locatio eſt linealis, aut arealis.

Linealis eſt, quæ per calculos in linea, pertica, aut margine abaci ſcacchiæ extenſos fit.

Sit Pertica a &, diviſa æqualiter in tot partes, quot numeros & calculos eam capere deſideras: verbi gratia in 16 partes ſi 16 calculos, aut 16 numeros eam capere velis: eritque decimusſextus numerus 32768.

Local arithmetic

Napier says that he had developed a method of doing arithmetic (even extracting roots) on a flat surface, a chessboard, by moving counters from square to square. He likens it to a game rather than work.

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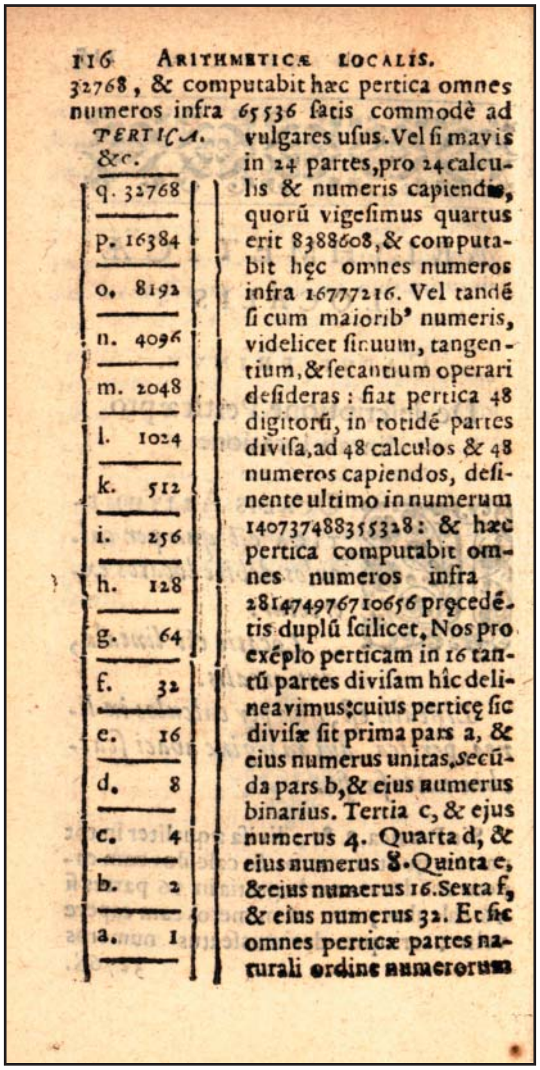
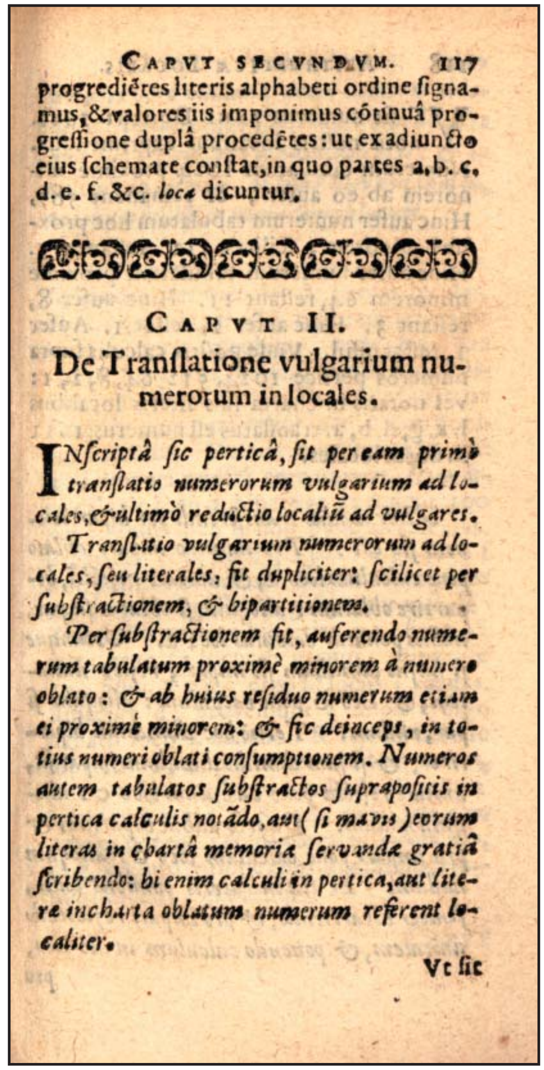


116 ARITHMETICÆ LOCALIS.

32768, & computabit hæc pertica omnes numeros infra 65536 satis commodè ad vulgares usus. Vel si mavis in 24 partes, pro 24 calculis & numeris capiendis, quorũ vigesimus quartus erit 8388608, & computabit hęc omnes numeros infra 16777216. Vel tandē si cum maiorib' numeris, videlicet sinuum, tangentium, & secantium operari desideras: fiat pertica 48 digitorũ, in totidē partes divisa, ad 48 calculos & 48 numeros capiendos, desinente ultimo in numerum 140737488355328: & hæc pertica computabit omnes numeros infra 281474976710656 procedē-tis duplũ scilicet. Nos pro exēplo perticam in 16 tantũ partes divisam hîc delineavimus; cuius perticę sic divisæ sit prima pars a, & eius numerus unitas, secũda pars b, & eius numerus binarius. Tertia c, & ejus numerus 4. Quarta d, & eius numerus 8. Quinta e, & eius numerus 16. Sexta f, & eius numerus 32. Et sic omnes perticæ partes naturali ordine numerorum



CAPUT SECUNDUM. 117

progrediētes literis alphabeti ordine signamus, & valores iis imponimus cōtinuâ progressione duplâ procedētes: ut ex adiuncto eius schemate constat, in quo partes a, b, c, d, e, f, &c. *loca* dicuntur.

CAPUT II.

De Translatione vulgarium numerorum in locales.

INscriptâ sic perticâ, sit per eam primò translatio numerorum vulgarium ad locales, & ultimò reductio localiũ ad vulgares.

Translatio vulgarium numerorum ad locales, seu literales, fit dupliciter: scilicet per substractionem, & bipartitionem.

Per substractionem fit, auferendo numerum tabulatum proximè minorem à numero oblato: & ab huius residuo numerum etiam ei proximè minorem: & sic deinceps, in totius numeri oblati consumptionem. Numeros autem tabulatos substractos suprapositis in pertica calculis notãdo, aut (si mavis) eorum literas in chartâ memoriæ servandæ gratiâ scribendo: hi enim calculi in pertica, aut literæ in charta oblatum numerum referent localiter.

Vt sic

He begins by saying that the only complication to this method is that one has to work with numbers of a different kind than the ordinary numbers. While working with these numbers (binary numbers, although he does not use that name) one has to perform elementary conversions to and from this system, but they are not difficult.

He begins by noting the various positions that a counter could occupy (on a line as he calls it) and these, and their values, are noted in the diagram on page 116.

If one uses a line of 16 units, then the last of them will have the value 32,768 which will be enough to calculate any value less than 65,536. By using a line of 24 units you can calculate any value less than 16,777,216.

If you need to calculate with larger numbers (e.g, sines, tangents and secants) then one of 48 units will allow values to be computed which are less than 281,474,976,710,656.

He labels each of these binary positions with a letter from *a* to *q*.

Chapter II: Changing ordinary numbers into location numbers.

He notes that the change from ordinary numbers to location numbers can be done via either subtraction or division.

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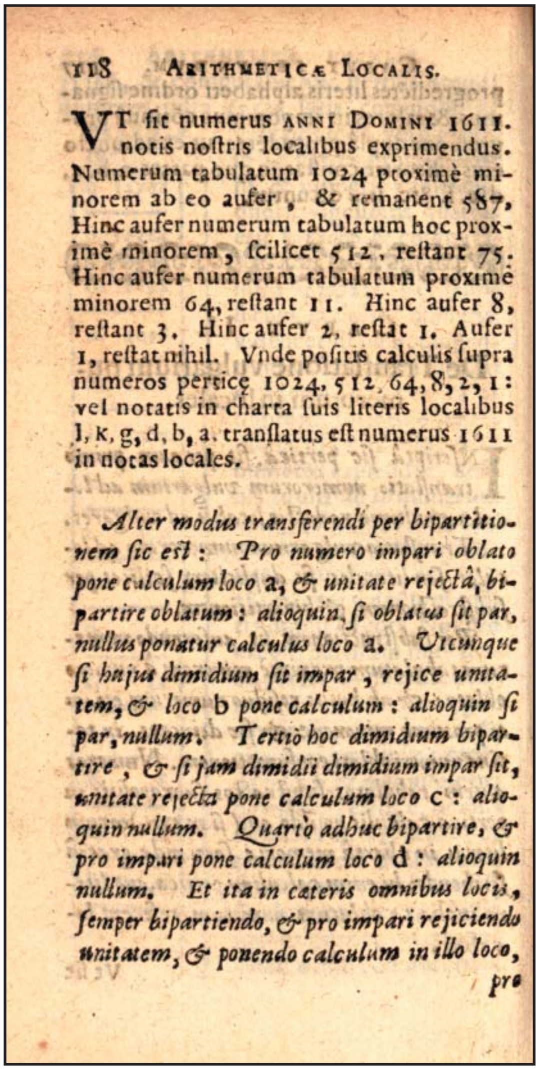
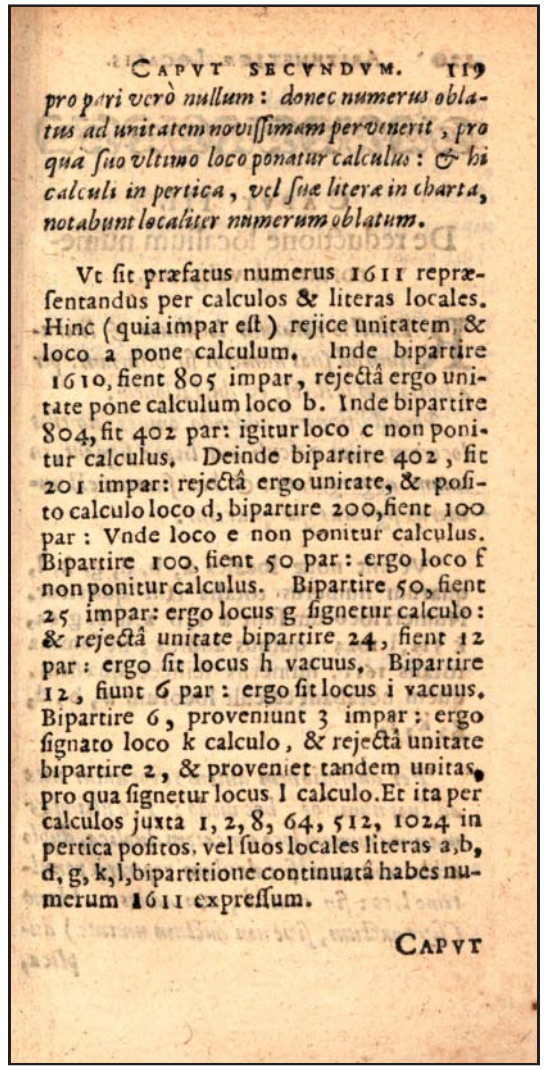


118 ARITHMETICÆ LOCALIS.

VT ſit numerus ANNI DOMINI 1611. notis noſtris localibus exprimendus. Numerum tabulatum 1024 proximè minorem ab eo aufer, & remanent 587. Hinc aufer numerum tabulatum hoc proximè minorem, ſcilicet 512, reſtant 75. Hinc aufer numerum tabulatum proximè minorem 64, reſtant 11. Hinc aufer 8, reſtant 3. Hinc aufer 2, reſtat 1. Aufer 1, reſtat nihil. Vnde poſitis calculis ſupra numeros perticę 1024, 512, 64, 8, 2, 1: vel notatis in charta ſuis literis localibus l, k, g, d, b, a. tranſlatus eſt numerus 1611 in notas locales.

Alter modus transferendi per bipartitionem ſic eſt: Pro numero impari oblato pone calculum loco a, *& unitate rejectâ, bipartire oblatum: alioquin, ſi oblatus ſit par, nullus ponatur calculus loco* a. *Vtcunque ſi hujus dimidium ſit impar, rejice unitatem, & loco* b *pone calculum: alioquin ſi par, nullum. Tertiò hoc dimidium bipartire, & ſi jam dimidii dimidium impar ſit, unitate rejectâ pone calculum loco* c*: alioquin nullum. Quartò adhuc bipartire, & pro impari pone calculum loco* d*: alioquin nullum. Et ita in cæteris omnibus locis, ſemper bipartiendo, & pro impari rejiciendo unitatem, & ponendo calculum in illo loco,*

pro



CAPVT SECVNDVM. 119

pro pari verò nullum: donec numerus oblatus ad unitatem noviſſimam pervenerit, pro qua ſuo ultimo loco ponatur calculus: & hi calculi in pertica, vel ſuæ literæ in charta, notabunt localiter numerum oblatum.

Vt ſit præfatus numerus 1611 repræſentandus per calculos & literas locales. Hinc (quia impar eſt) rejice unitatem, & loco a pone calculum. Inde bipartire 1610, fient 805 impar, rejectâ ergo unitate pone calculum loco b. Inde bipartire 804, fit 402 par: igitur loco c non ponitur calculus. Deinde bipartire 402, fit 201 impar: rejectâ ergo unitate, & poſito calculo loco d, bipartire 200, fient 100 par: Vnde loco e non ponitur calculus. Bipartire 100, fient 50 par: ergo loco f non ponitur calculus. Bipartire 50, fient 25 impar: ergo locus g ſignetur calculo: & rejectâ unitate bipartire 24, fient 12 par: ergo ſit locus h vacuus. Bipartire 12, fiunt 6 par: ergo ſit locus i vacuus. Bipartire 6, proveniunt 3 impar: ergo ſignato loco k calculo, & rejectâ unitate bipartire 2, & proveniet tandem unitas, pro qua ſignetur locus l calculo. Et ita per calculos juxta 1, 2, 8, 64, 512, 1024 in pertica poſitos, vel ſuos locales literas a, b, d, g, k, l, bipartitione continuatâ habes numerum 1611 expreſſum.

CAPVT

As an example of converting to location numbers he shows how repeated subtractions of numbers on this binary line will change 1611 to counters on locations *l, k, g, d, b* and *a* (or places representing 1024, 512, 64, 8, 2 and 1).

He describes the algorithm for the division method as follows:

If the number is odd put a counter on position *a*, subtract 1 from the number and then divide it by 2. If the number is even leave position *a* empty.

If this half number is odd then subtract 1 and place a counter in position *b*, if it is even leave *b* empty.

Proceed, as above, by dividing the number by 2, if the result is odd subtract 1 and put down a counter, if the result is even do nothing.

When the result is reduced to 1, put a counter in that position and the process terminates.

He again illustrates the process with the number 1611.

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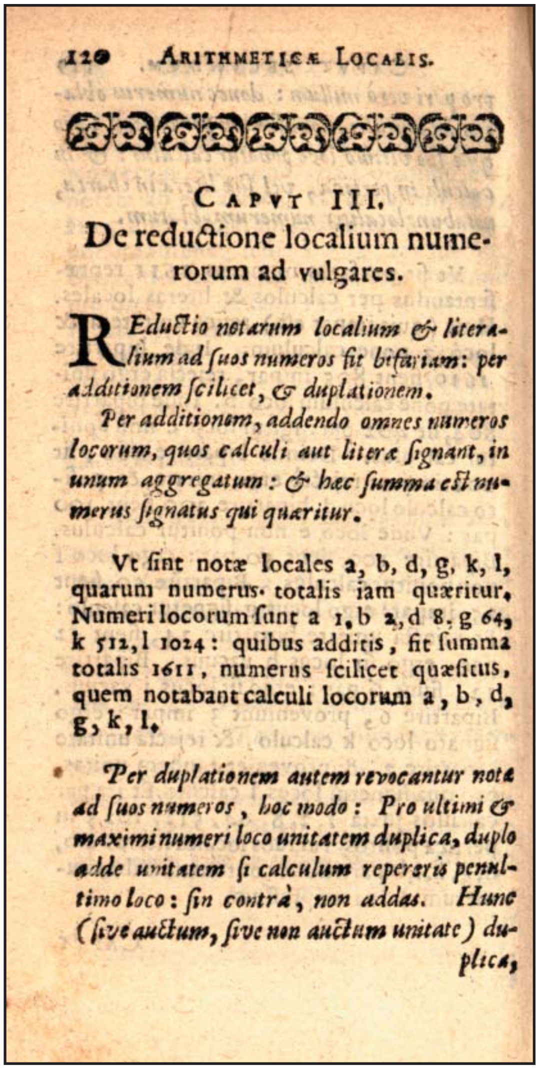

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CAPVT III.

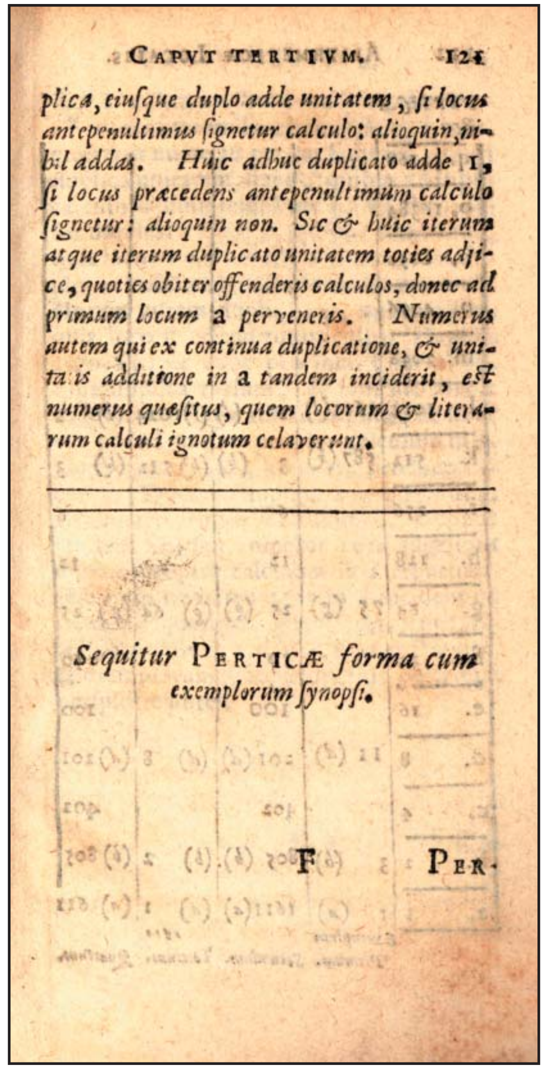
De reductione localium numerorum ad vulgares.

Reductio notarum localium & literalium ad suos numeros fit bifariam: per additionem scilicet, & duplationem.

Per additionem, addendo omnes numeros locorum, quos calculi aut literæ signant, in unum aggregatum: & hæc summa est numerus signatus qui quæritur.

Vt sint notæ locales a, b, d, g, k, l, quarum numerus totalis iam quæritur. Numeri locorum sunt a 1, b 2, d 8, g 64, k 512, l 1024: quibus additis, fit summa totalis 1611, numerus scilicet quæsitus, quem notabant calculi locorum a, b, d, g, k, l.

Per duplationem autem revocantur notæ ad suos numeros, hoc modo: Pro ultimi & maximi numeri loco unitatem duplica, duplo adde unitatem si calculum reperєris penultimo loco: sin contrà, non addas. Hunc (sive auctum, sive non auctum unitate) duplica,



CAPVT TERTIVM. 121

plica, eiusque duplo adde unitatem, si locus antepenultimus signetur calculo: alioquin, nihil addas. Huic adhuc duplicato adde 1, *si locus præcedens antepenultimum calculo signetur: alioquin non. Sic & huic iterum atque iterum duplicato unitatem toties adjice, quoties obiter offenderis calculos, donec ad primum locum* a *perveneris. Numerus autem qui ex continua duplicatione, & unitatis additione in* a *tandem inciderit, est numerus quæsitus, quem locorum & literarum calculi ignotum celaverunt.*

Sequitur PERTICÆ *forma cum exemplorum synopsi.*

F PER-

Chapter III: changing location number to ordinary numbers.

This short section indicates that one can recover the usual numbers from the location representation by adding up those values that have a counter on the position. Alternatively it can be done by a process of doubling analogous to that done by division to convert it to the location representation.

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Pertica &c.	Exemplum Primum.	Secundum.	Tertium.	Quartum.
q. 32768				
p. 16384				
o. 8192				
n. 4096				
m. 2048				
l. 1024	1611 (l)	1 (l)	(l) 1024	(l) 1
k. 512	587 (k)	3 (k)	(k) 512	(k) 3
i. 256		6		6
h. 128		12		12
g. 64	75 (g)	25 (g)	(g) 64	(g) 25
f. 32		50		50
e. 16		100		100
d. 8	11 (d)	201 (d)	(d) 8	(d) 201
c. 4		402		402
b. 2	3 (b)	805 (b)	(b) 2	(b) 805
a. 1	1 (a)	1611 (a)	(a) 1	(a) 1611
			1611	

Vt adhuc per hunc modum, ſuperiorum notarum localium l, k, g, d, b, a, quæratur numerus per eas ſignatus. Pro l duplica unitatem, fient 2: quibus adde pro calculo k unitatem, fient 3. Quibus rurſus duplicatis fient 6, quibus nihil adde, quia locus i vacuus eſt. Horum etiam duplo 12 nihil adde, quia locus h vacuus eſt. Horum autem duplo, 24, unum adde, quia loco g offenditur calculus. Duplica ergo 25, & fiunt pro loco f vacuo 50. Quæ duplica rurſum, & ſunt pro loco e vacuo 100. Quæ adhuc duplica, & fiunt 201 additâ unitate propter calculum in d. Duplica ergo 201, fiunt 402 pro loco vacuo c. Quorum duplum 804 augendum eſt unitate propter calculum b. Duplica tandem 805 fiet numerus 1610 augendus unitate propter calculum in a repertum. Hic ergo numerus 1611 in a incidens eſt numerus quæſitus, quem celaverunt calculi, & literæ a, b, d, g, k, l: ut ex Pertica, & exemplorum ſynopſi ſuperiùs deſcriptis perſpicere potes.

This diagram is simply a representation for the four previous examples and shows the workings of converting and reconverting the number 1611.

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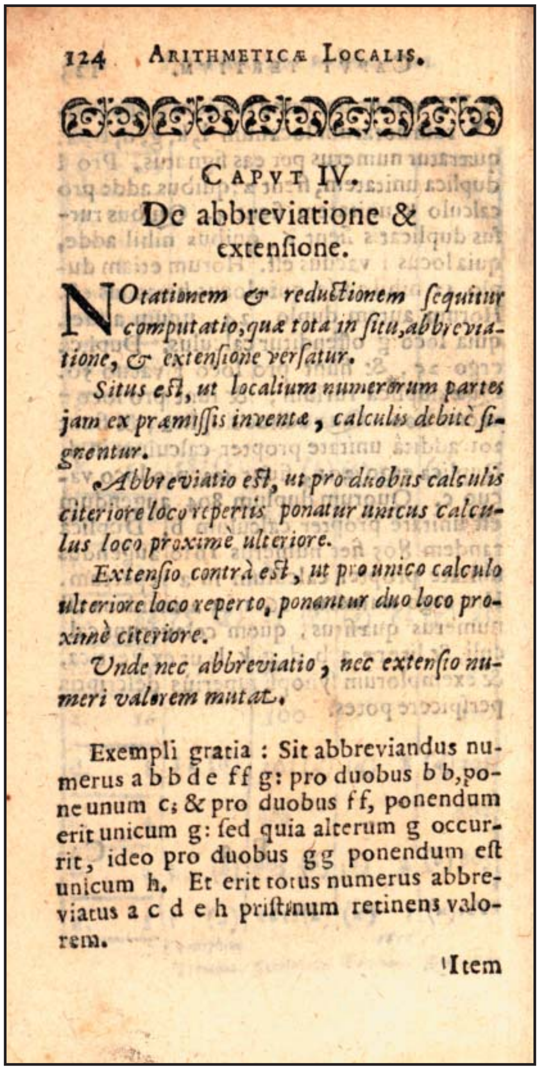

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CAPVT IV.

De abbreviatione & extensione.

NOtationem & reductionem sequitur computatio, quæ tota in situ, abbreviatione, & extensione versatur.

Situs est, ut localium numerorum partes jam ex præmissis inventæ, calculis debitè signentur.

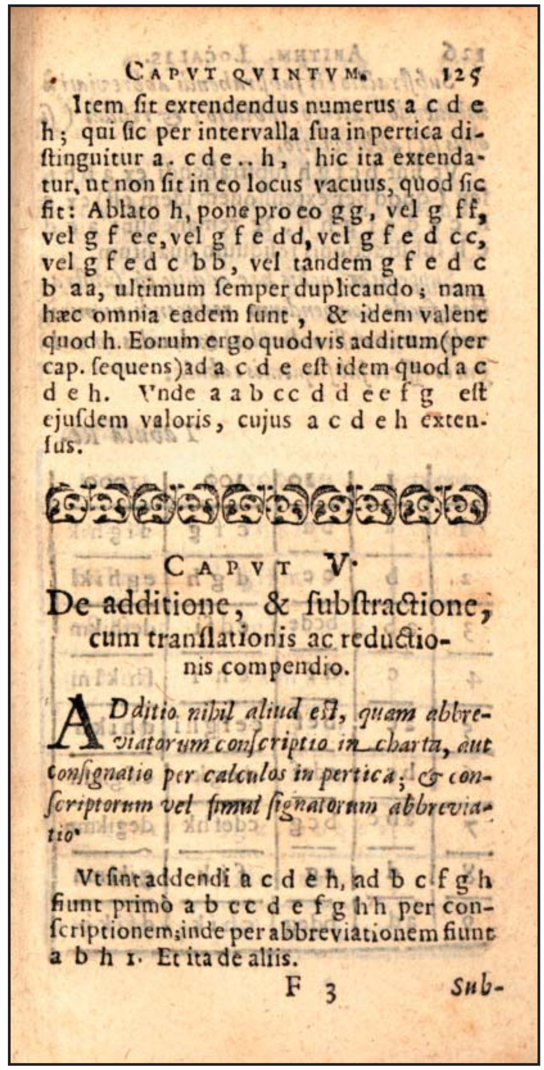
Abbreviatio est, ut pro duobus calculis citeriore loco repertis ponatur unicus calculus loco proximè ulteriore.

Extensio contrà est, ut pro unico calculo ulteriore loco reperto, ponantur duo loco proximè citeriore.

Vnde nec abbreviatio, nec extensio numeri valorem mutat.

Exempli gratia: Sit abbreviandus numerus a b b d e ff g: pro duobus b b, pone unum c; & pro duobus ff, ponendum erit unicum g: sed quia alterum g occurrit, ideo pro duobus g g ponendum est unicum h. Et erit totus numerus abbreviatus a c d e h pristinum retinens valorem.

Item



CAPVT QVINTVM. 125

Item sit extendendus numerus a c d e h; qui sic per intervalla sua in pertica distinguitur a . c d e . . h, hic ita extendatur, ut non sit in eo locus vacuus, quod sic fit: Ablato h, pone pro eo g g, vel g ff, vel g f ee, vel g f e d d, vel g f e d cc, vel g f e d c b b, vel tandem g f e d c b aa, ultimum semper duplicando; nam hæc omnia eadem sunt, & idem valent quod h. Eorum ergo quodvis additum (per cap. sequens) ad a c d e est idem quod a c d e h. Vnde a a b cc d d ee f g est ejusdem valoris, cujus a c d e h extensus.

CAPVT V.

De additione, & substractione, cum translationis ac reductionis compendio.

ADditio nihil aliud est, quam abbreviatorum conscriptio in charta, aut consignatio per calculos in pertica; & conscriptorum vel simul signatorum abbreviatio.

Vt sint addendi a c d e h, ad b c f g h fiunt primò a b c c d e f g h h per conscriptionem, inde per abbreviationem fiunt a b h i. Et ita de aliis.

F 3 Sub-

Chapter IV: Abbreviations and extensions.

Abbreviation means that you may replace any two counters in a given position by one on the next higher position.

Extension means that you may replace any single counter in a position by two counters in the next lower position and repeating this operation until there are no blank spaces (each lower position contains a counter).

Neither operation changes the value of the number being represented.

As an example he shows that a number *abbdeffg* may be abbreviated by replacing the two *b*s with a single *c*, the two *f*s by a single *g* (resulting now in there being two *g*s) and the two *g*s by a single *h*. The resulting number (*acdeh*) has the same value as the one you started with (*abbdeffg*). Similarly this original number (*abbdeffg*) could be extended by replacing a higher counter with two lower ones to ultimately yield *abbccddeefg*.

This extending and abbreviating would not have been unknown to Napier's readers because similar processes were used on the European table abacus of the time—where two counters representing values of 5 could be replaced by one representing 10, two 50s by a 100, etc. (see any of these Tomash reproduction files of works on the table abacus for further information).

Chapter V: Addition and Subtraction.

Addition is accomplished by setting the position counters for two numbers side by side and then abbreviating the resulting total set of counters. He give the example of the numbers *acdeh* added to *bcfgh* which, when grouped together, are *abccdefghh* and when this is abbreviated it becomes *abhi*.

Substractio est substrahendi abbreviati in minuendo extenso sublatio; & residui (si opus sit) abbreviatio.

Vt sint b c f g h substrahendi ex a b h i, seu (quod per extensionem idem est) ex a b c c d e f g h h, & remanebunt a c d e h substractionis residuum quæsitum.

Suppeditat nobis hæc additio & substractio facile compendium reducendi numeros vulgares in nostros locales, & locales in vulgares officio subsequentis tabulæ.

Vt sint 3783 reducenda ad nostros locales numeros. Quære primò in Tabula 3000, in communi angulo inter 3 & 1000, & offendes d e f h i k m. Quære item 700 inter 7 & 100, & offendes c d e f h k. Quęre tertiò 80 in communi angulo inter 8 & 10, & reperies e g. Quęre tandem 3, & reperies pro iis a b in communi angulo inter 3 & 1. Has quatuor summas (ex pręmissis) adde, & fient a b c g h k l m pro numero 3783.

Tabula Reductionis.

	1	10	100	1000	10000	100000	1000000
1	a	bd	c f g	dfghik	eiklo	fhklqr	gkprstv
2	b	ce	dgh	eghikl	fklmp	gilmrs	
3	ab	bcde	cdfi	defhikm	efilnop	fghiknqt	
4	c	df	ehi	fhiklm	glmnq	hkmnst	
5	ac	bef	cefghi	dhikn	egikpq	fioqrst	
6	bc	cdef	degk	efgikln	fgkmopq	ghiklorv	
7	abc	bcg	cdefhk	degikmn	efginr	fgklmoqsv	
8	d	eg	fik	giklmn	hmnor	ilnotv	
9	ad	bdeg	chik	dfiko	ehiklmnpr	fhikmnoqrtv	

Subtraction is performed by placing the two numbers adjacent and, making sure that the larger number is in extended form (i.e, there are no blank spaces without counters), remove from it every counter than matches one in the abbreviated form of the lesser number and, if needed, abbreviating the result.

The table is a listing of base 10 integers and their corresponding position (binary) numbers in Napier's alphabetic notation (see diagram on page 122). This provides a simple way of converting to and from the binary positional notation.

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Contrà, ſint reducenda a b c g h k l m, ſeu (quæ per extenſionem eadem ſunt) a b c d d e f g g h h i k k m, ad numerum vulgarem.

Aufer hinc maximum numerum localem tabulatum qui auferri poſſit, ſcilicet d e f h i k m (pro 3000) & ſuperſunt a b c d g g h k. Ex quibus (per CAP. 4. hujus) extenſis ad a b c d e e f g h k, auferatur tabulatus numerus localis quàm maximus c d e f h k, (qui 700nis reſpondet) & remanent a b e g. à quibus aufer tabulatum e g, (qui 80 reſpondet) & ſuperſunt a b, quibus tria in Tabula reſpondent.

Numeri itaque præfati inventi, ſunt 3000, 700, 80, 3, ſeu conjunctì 3783, qui eſt numerus petitus reſpondens literalibus calculis a b c g h k l m.

ADMONITIO.

Poteſt etiam per hanc Perticam multiplicatio & diviſio perfici: ſed quia hæc opera lucidius multò & facilius expediuntur per arealem locationem, quæ fit in alveo ſcacchiæ utramque locationem complectente, quàm quæ per Perticam ſolam fit: ad arealem locationem, ejuſque uſum in expediendis multiplicationibus, diviſionibus, & radicum extractionibus, ſermonem convertamus.

CAPVT

CAPVT VI.

De deſcriptione abaci, vel alvei, pro locatione areali.

AREALIS *numerorum locatio, eſt eorundem deſignatio per calculos in areolis & cancellis alvei ſcacchorum ſeu latrunculorum, vel alterius ſimilis quadratæ Tabulæ depoſitos.*

SIT Tabula hæc quadrata ♈ ♉ ♊ ♋, angulus tibi proximus ♈, angulus ſiniſter ♉, angulus à te remotiſsimus ♊, angulus dexter ♋. Dividatur latus ♈ ♉ in quotvis partes, utpote in 18, 24, vel in plures, ſecundum numeros & calculos quos abacum capere deſideras: nos ſequenti ſchemate illud in 24 dividimus. Dividantur etiam latus ♈ ♋, & reliqua latera in totidem partes; & ductis lineis à latere ♈ ♉ ad latus ♊ ♋, & à latere ♈ ♋ ad latus ♉ ♊, per ſingula diviſionum puncta, habebis tabulam diviſam arealiter in 576 areolas quadratas. Dextrum latus ab ♈ in ♋, & à ♋ in ♊, ſignabis literis abecedarii, & numeris duplo progreſſu, ut in pertica; & ubi abecedarium latinum

F 5 deficit,

Chapter VI: Description of the board (abacus) in two dimensions.

Create a board, similar to that illustrated between pages 131 and 132, with as many rows and columns as you will need for the size of numbers to be considered—here Napier uses 24 rows and columns. The illustration mentioned was bound after page 131 on a piece of paper with the notation *Pro pag. 130* uppermost. Napier clearly says that the corner labeled ♈ should be positioned closest to you so that is the way we represent it here (i.e., rotated 45 degrees from the original orientation).

Each row and column should be labeled with a set of binary numbers and with alphabetic symbols (he suggests continuing with the Greek alphabet if you run out of the Roman letters).

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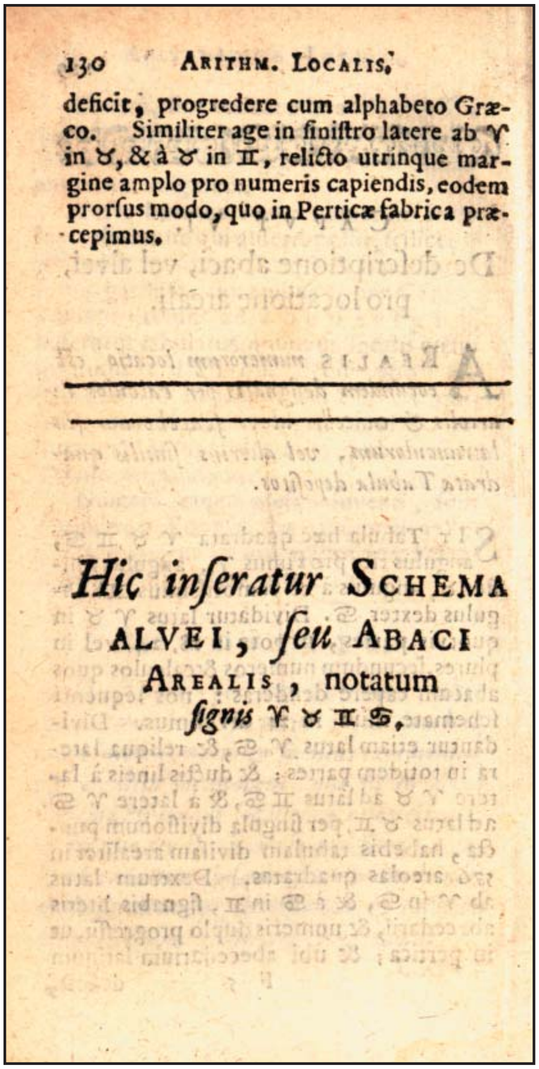
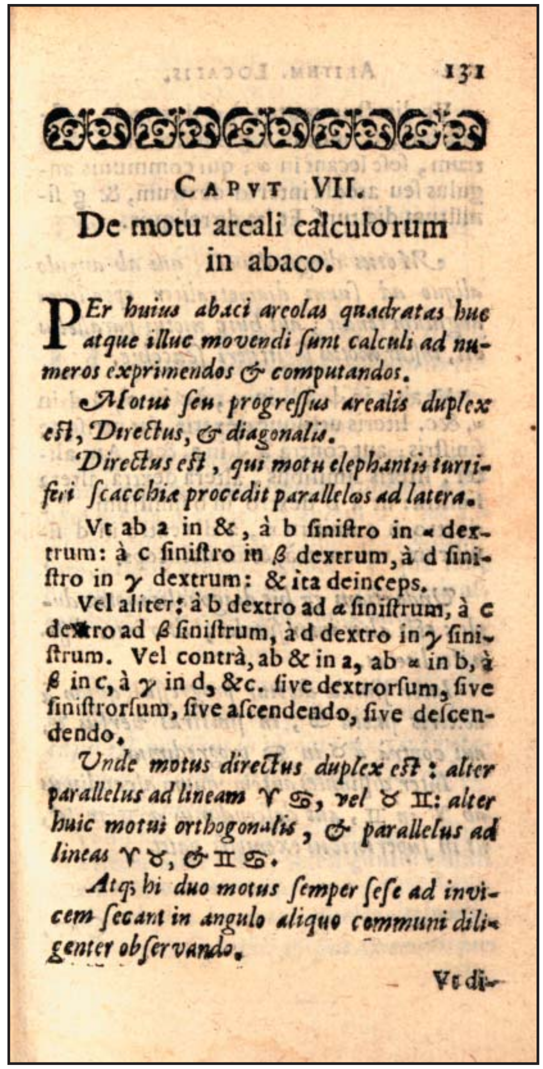

130 ARITHM. LOCALIS.

deficit, progredere cum alphabeto Græco. Similiter age in ſiniſtro latere ab ♈ in ♉, & à ♉ in ♊, relicto utrinque margine amplo pro numeris capiendis, eodem prorſus modo, quo in Perticæ fabrica præcepimus.

Hic inſeratur SCHEMA ALVEI, ſeu ABACI AREALIS, notatum ſignis ♈ ♉ ♊ ♋.


ARITHM. LOCALIS. 131

CAPVT VII.

De motu areali calculorum in abaco.

PEr huius abaci areolas quadratas huc atque illuc movendi ſunt calculi ad numeros exprimendos & computandos.

Motus ſeu progreſſus arealis duplex eſt, Directus, & diagonalis.

Directus eſt, qui motu elephantis turriferi ſcacchiæ procedit parallelos ad latera.

Vt ab a in &, à b ſiniſtro in α dextrum: à c ſiniſtro in β dextrum, à d ſiniſtro in γ dextrum: & ita deinceps.

Vel aliter: à b dextro ad α ſiniſtrum, à c dextro ad β ſiniſtrum, à d dextro in γ ſiniſtrum. Vel contrà, ab & in a, ab α in b, à β in c, à γ in d, &c. ſive dextrorſum, ſive ſiniſtrorſum, ſive aſcendendo, ſive deſcendendo.

Vnde motus directus duplex eſt: alter parallelus ad lineam ♈ ♋, vel ♉ ♊: alter huic motui orthogonalis, & parallelus ad lineas ♈ ♉, & ♊ ♋.

Atq; hi duo motus ſemper ſeſe ad invicem ſecant in angulo aliquo communi diligenter obſervando.

Vt di-

Chapter VII: moving the counters on the abacus.

Computations are accomplished by moving counters on this board. The movement is of two types, direct and diagonal. Direct movement is parallel to the sides of the board while diagonal movement is up and down or left and right (i.e., following the diagonals, like a bishop on the square orientation of the chess board). Diagonal movement can be from two common letters (e.g., *f*-right to *f*-left) or directly up or down the board (e.g., from *a* to *ψ* or vice versa etc.).

In direct movement (parallel to the sides of the board) there are always places where the two directions meet and these will be important. For example, direct movement from *d* on the right hand side to *γ* on the left and from *g* on the left to *ζ* on the right will intersect at the square marked *ω*. This point is said to be common to *d*-right and *g*-left.

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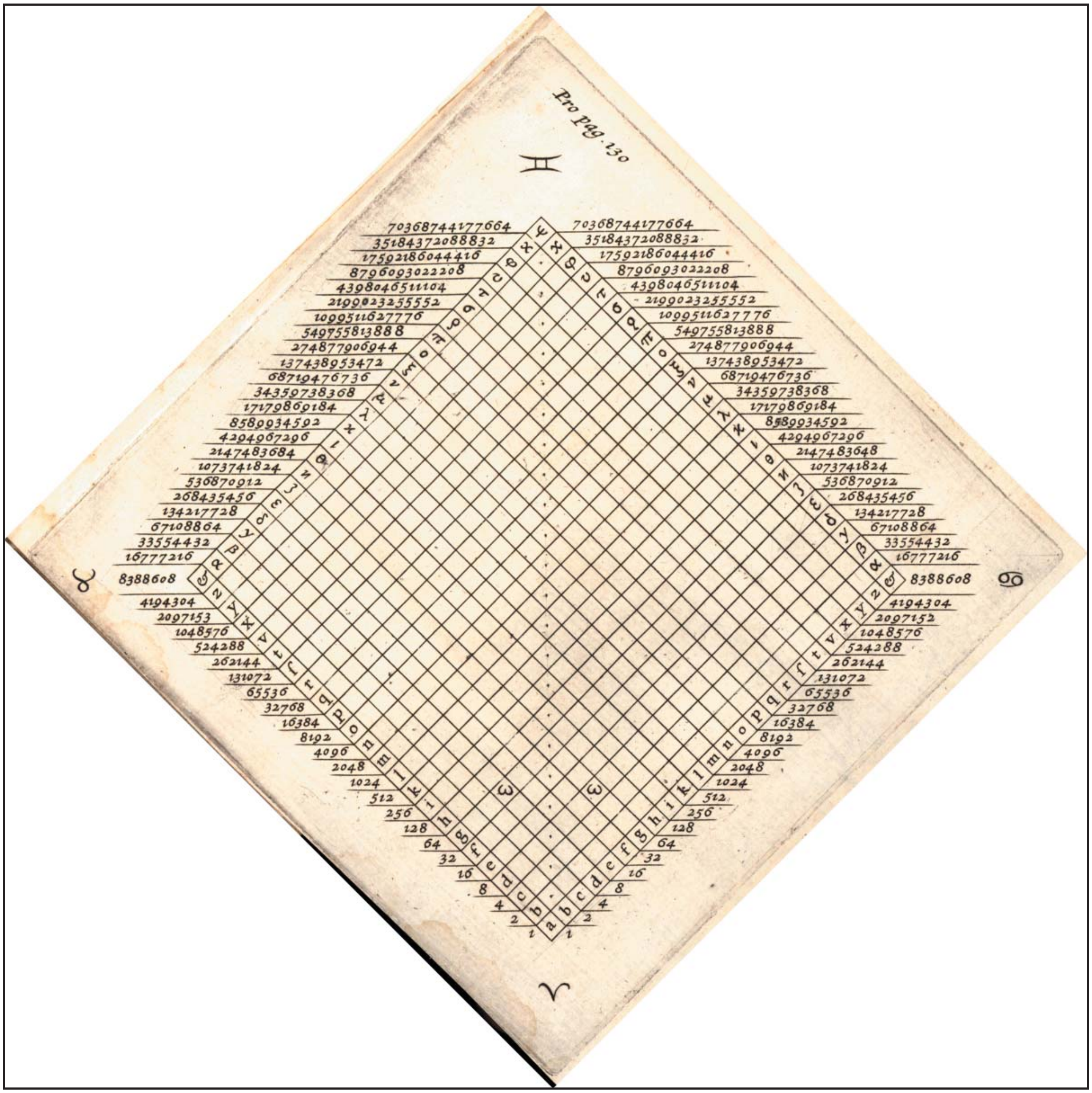

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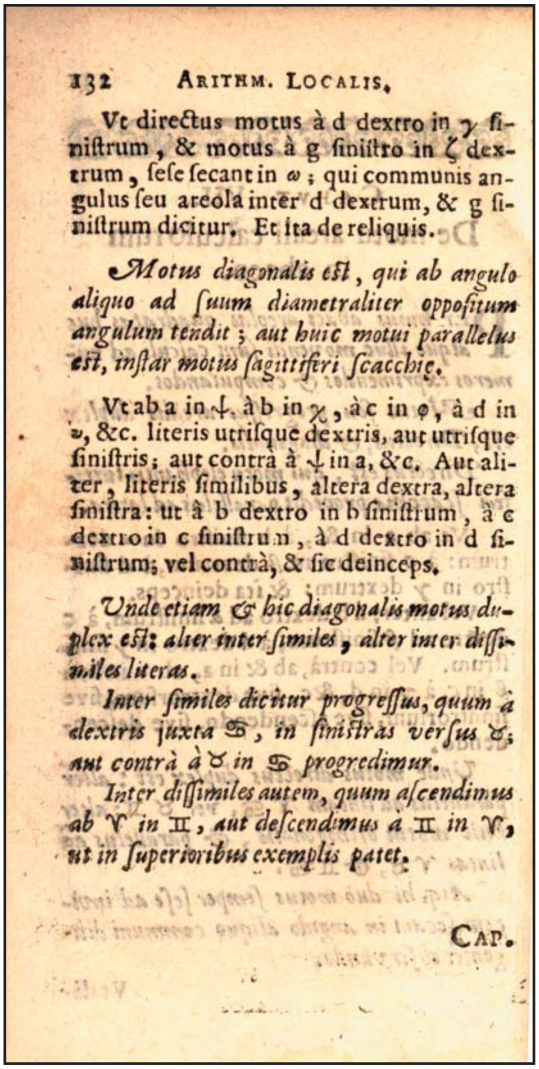


132 ARITHM. LOCALIS.

Vt directus motus à d dextro in γ sinistrum, & motus à g sinistro in ζ dextrum, sese secant in ω; qui communis angulus seu areola inter d dextrum, & g sinistrum dicitur. Et ita de reliquis.

Motus diagonalis est, qui ab angulo aliquo ad suum diametraliter oppositum angulum tendit; aut huic motui parallelus est, instar motus sagittiferi scacchiç.

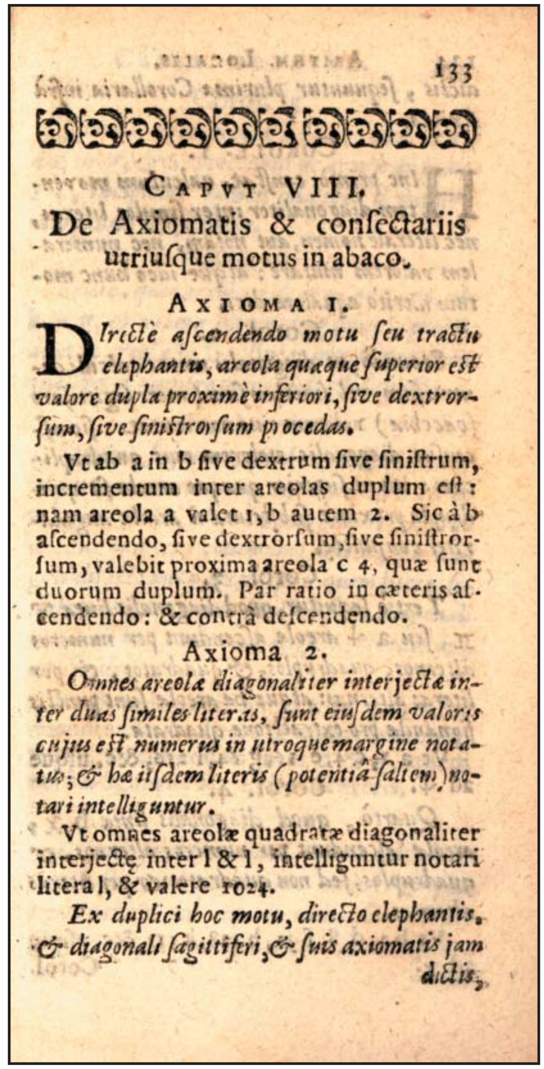
Vt ab a in ⸸, à b in χ, à c in φ, à d in υ, &c. literis utrisque dextris, aut utrisque sinistris; aut contra à ⸸ in a, &c. Aut aliter, literis similibus, altera dextra, altera sinistra: ut à b dextro in b sinistrum, à c dextro in c sinistrum, à d dextro in d sinistrum; vel contrà, & sic deinceps.

Unde etiam & hic diagonalis motus duplex est: alter inter similes, alter inter dissimiles literas.

Inter similes dicitur progressus, quum à dextris juxta ♋, in sinistras versus ♉; aut contrà à ♉ in ♋ progredimur.

Inter dissimiles autem, quum ascendimus ab ♈ in ♊, aut descendimus à ♊ in ♈, ut in superioribus exemplis patet.

CAP.



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CAPVT VIII.
De Axiomatis & consectariis utriusque motus in abaco.

AXIOMA I.

Directè ascendendo motu seu tractu elephantis, areola quaque superior est valore dupla proximè inferiori, sive dextrorsum, sive sinistrorsum procedas.

Vt ab a in b sive dextrum sive sinistrum, incrementum inter areolas duplum est: nam areola a valet 1, b autem 2. Sic à b ascendendo, sive dextrorsum, sive sinistrorsum, valebit proxima areola c 4, quæ sunt duorum duplum. Par ratio in cæteris ascendendo: & contrà descendendo.

Axioma 2.

Omnes areolæ diagonaliter interjectæ inter duas similes literas, sunt eiusdem valoris cujus est numerus in utroque margine notatus; & hæ iisdem literis (potentiâ saltem) notari intelliguntur.

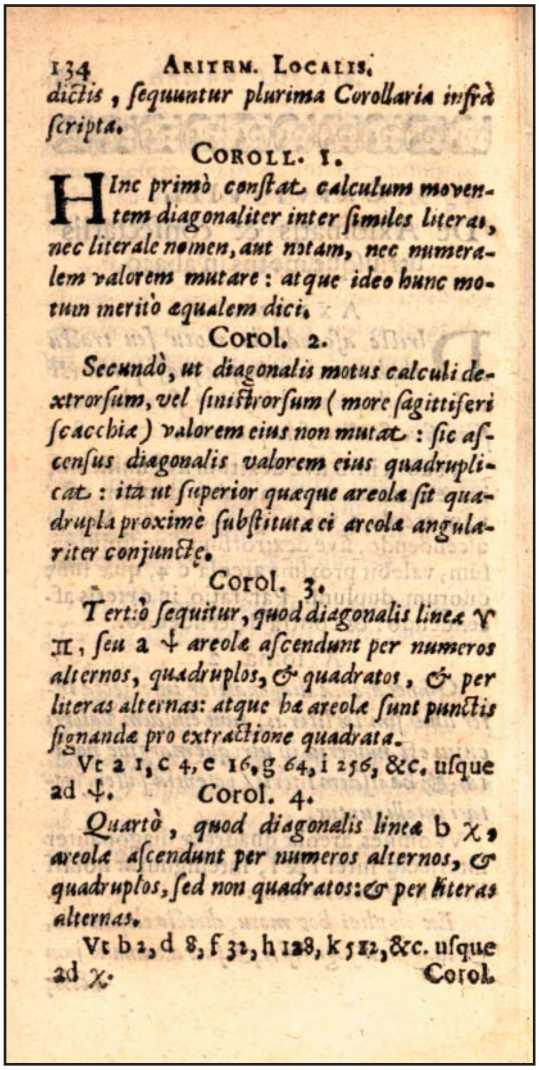
Vt omnes areolæ quadratæ diagonaliter interjectę inter l & l, intelliguntur notari litera l, & valere 1024.

Ex duplici hoc motu, directo elephantis, & diagonali sagittiferi, & suis axiomatis jam dictis,

Chapter VIII: Rules for each type of movement on the abacus (board).

In direct movement (parallel to the sides of the board) a movement of one square doubles the value.
In diagonal movement (up and down or side to side on the diagonals), all squares have equal value and are assumed to be labeled with the same letters as at their end points.

Moving a counter to a square immediately above it will multiply the value by 4 (i.e., from the square *a,c* to the square *a,e* changes the value from 4 to 16).



134 ARITHM. LOCALIS.

dictis, sequuntur plurima Corollaria infrà scripta.

COROLL. I.

Hinc primò constat calculum moventem diagonaliter inter similes literas, nec literale nomen, aut notam, nec numeralem valorem mutare: atque ideo hunc motum meritò æqualem dici.

Corol. 2.

Secundò, ut diagonalis motus calculi dextrorsum, vel sinistrorsum (more sagittiferi scacchiæ) valorem eius non mutat: sic ascensus diagonalis valorem eius quadruplicat: ita ut superior quæque areola sit quadrupla proxime substitutæ ei areolæ angulariter conjunctæ.

Corol. 3.

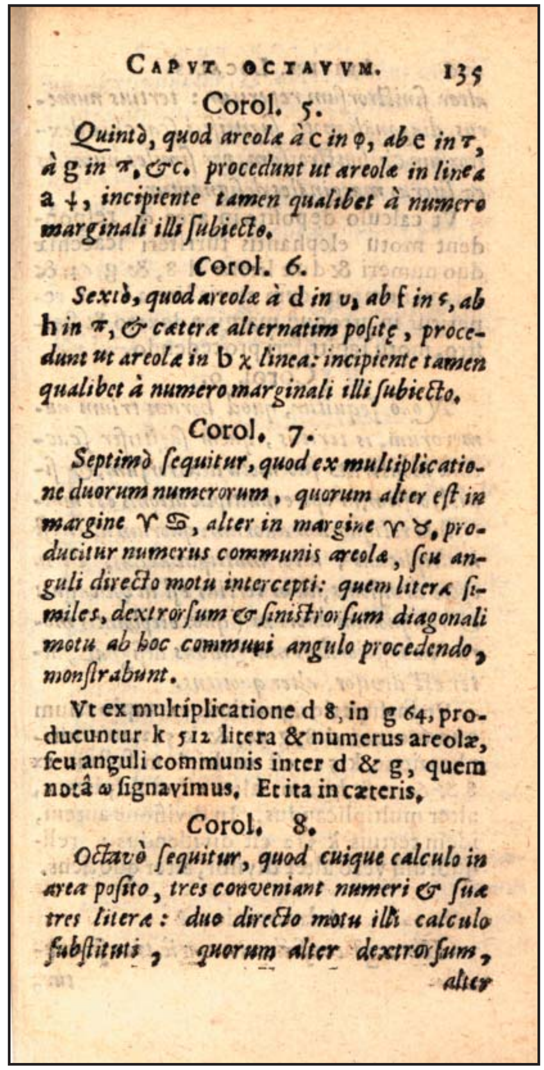
Tertiò sequitur, quod diagonalis lineæ ♈ π, seu a ψ areolæ ascendunt per numeros alternos, quadruplos, & quadratos, & per literas alternas: atque hæ areolæ sunt punctis signandæ pro extractione quadrata.

Vt a 1, c 4, e 16, g 64, i 256, &c. usque ad ψ.

Corol. 4.

Quartò, quod diagonalis lineæ b χ, areolæ ascendunt per numeros alternos, & quadruplos, sed non quadratos: & per literas alternas.

Vt b 2, d 8, f 32, h 128, k 512, &c. usque ad χ. Corol.



CAPVT OCTAVVM. 135

Corol. 5.

Quintò, quod areolæ à c in φ, ab e in τ, à g in π, &c. procedunt ut areolæ in lineâ a ψ, incipiente tamen qualibet à numero marginali illi subiecto.

Corol. 6.

Sextò, quod areolæ à d in υ, ab f in ς, ab h in π, & cæteræ alternatim positæ, procedunt ut areolæ in b χ lineæ: incipiente tamen qualibet à numero marginali illi subiecto.

Corol. 7.

Septimò sequitur, quod ex multiplicatione duorum numerorum, quorum alter est in margine ♈ ♋, alter in margine ♈ ♉, producitur numerus communis areolæ, seu anguli directo motu intercepti: quem literæ similes, dextrorsum & sinistrorsum diagonali motu ab hoc communi angulo procedendo, monstrabunt.

Vt ex multiplicatione d 8, in g 64, producuntur k 512 litera & numerus areolæ, seu anguli communis inter d & g, quem notâ ω signavimus. Et ita in cæteris.

Corol. 8.

Octavò sequitur, quod cuique calculo in area posito, tres conveniant numeri & suæ tres literæ: duo directo motu illi calculo substituti, quorum alter dextrorsum, alter

All squares up the middle from *a* to *ψ* are each 4 times the previous one and are all perfect squares (i.e., 1, 4, 16, 64, etc.). They are marked by a dot so that they may be easily located when calculating square roots.

Squares on the vertical line beside the dots (from *b* to χ) are again 4 times the previous one (2, 8, 32 etc) but start with the number shown on the margin.

A number set up on one margin (say ♈to ♉) multiplied by another set on the other margin (♈ to ♋) is the square at the intersection of the two values. Napier gives an example of a counter on *d* (8) and another on *g* (64) have a product of 512 (the intersection of the two, the right square marked *ω*). For any such internal square, there are three numbers that are associated with it: the one designating the diagonals on which it is positioned (*d* or 8 and *g* or 64) and the one designating the value in the margin horizontal to its position (*k* or 512).

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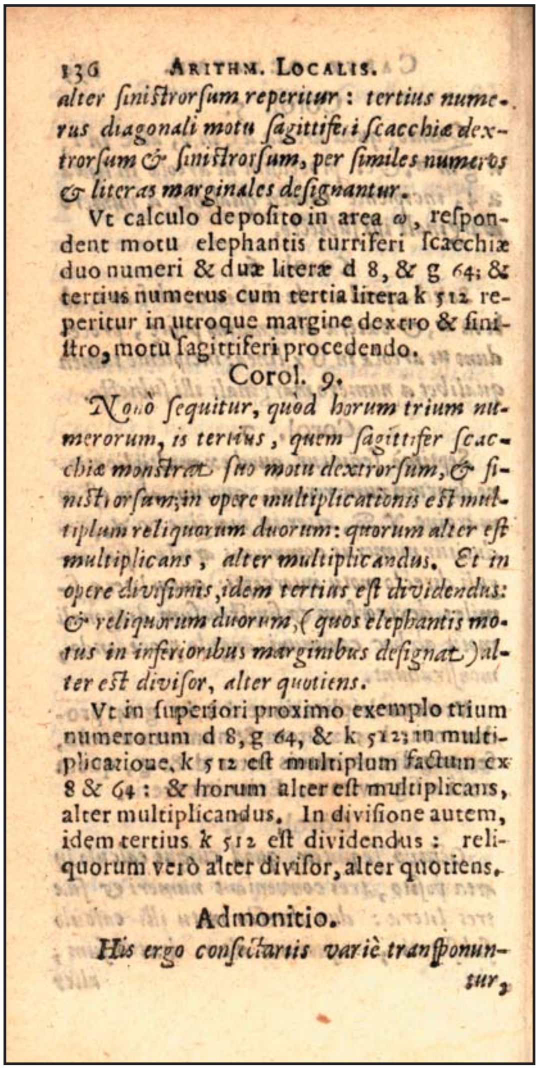
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alter sinistrorsum reperitur: tertius numerus diagonali motu sagittiferi scacchiæ dextrorsum & sinistrorsum, per similes numeros & literas marginales designantur.

Vt calculo deposito in area ω, respondent motu elephantis turriferi scacchiæ duo numeri & duæ literæ d 8, & g 64; & tertius numerus cum tertia litera k 512 reperitur in utroque margine dextro & sinistro, motu sagittiferi procedendo.

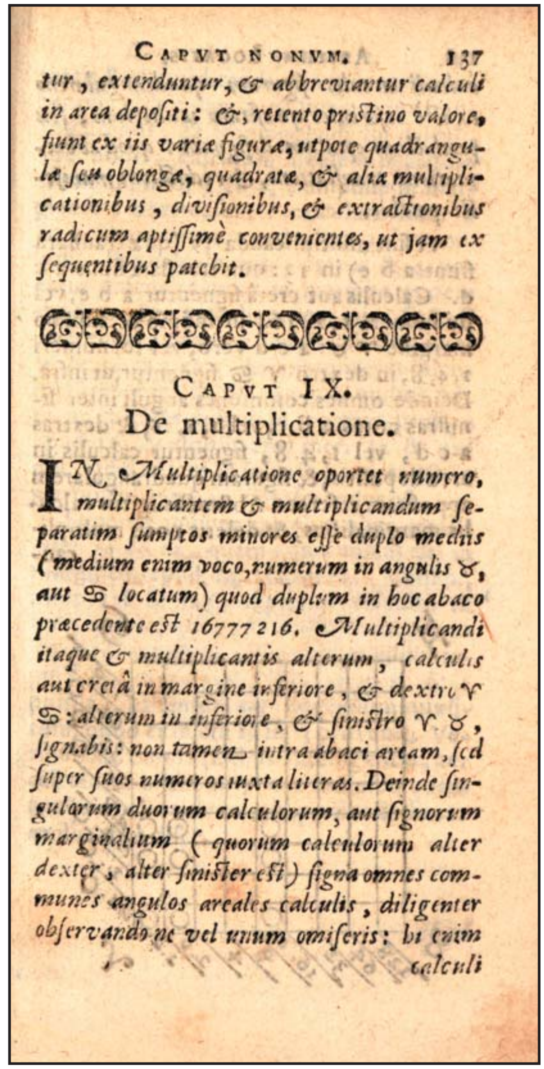
Corol. 9.

Nono sequitur, quod horum trium numerorum, is tertius, quem sagittifer scacchiæ monstrat suo motu dextrorsum, & sinistrorsum; in opere multiplicationis est multiplum reliquorum duorum: quorum alter est multiplicans, alter multiplicandus. Et in opere divisionis, idem tertius est dividendus: & reliquorum duorum, (quos elephantis motus in inferioribus marginibus designat) alter est divisor, alter quotiens.

Vt in superiori proximo exemplo trium numerorum d 8, g 64, & k 512; in multiplicatione, k 512 est multiplum factum ex 8 & 64: & horum alter est multiplicans, alter multiplicandus. In divisione autem, idem tertius k 512 est dividendus: reliquorum verò alter divisor, alter quotiens.

Admonitio.

His ergo consectariis variè transponuntur,


CAPUT NONUM. 137

tur, extenduntur, & abbreviantur calculi in area depositi: &, retento pristino valore, fiunt ex iis variæ figuræ, utpote quadrangulæ seu oblongæ, quadratæ, & aliæ multiplicationibus, divisionibus, & extractionibus radicum aptissimè convenientes, ut jam ex sequentibus patebit.

CAPUT IX.

De multiplicatione.

IN Multiplicatione oportet numero, multiplicantem & multiplicandum separatim sumptos minores esse duplo medijs (medium enim voco, numerum in angulis ♉, aut ♋ locatum) quod duplum in hoc abaco præcedente est 16777216. Multiplicandi itaque & multiplicantis alterum, calculis aut cretâ in margine inferiore, & dextro ♈ ♋: alterum in inferiore, & sinistro ♈ ♉, signabis: non tamen intra abaci aream, sed super suos numeros iuxta literas. Deinde singulorum duorum calculorum, aut signorum marginalium (quorum calculorum alter dexter, alter sinister est) signa omnes communes angulos areales calculis, diligenter observando ne vel unum omiseris: hi enim calculi

Chapter IX: Multiplication.

Make sure that both of your numbers can be represented by counters being placed lower than half way up the board (i.e., between ♈to ♉ on one side and ♈ to ♋ on the other). On the board shown in his diagram each number must be less than or equal to 16,777,216.

Place counters (or otherwise mark) the positions in the margins for each number. A counter is to be placed at the intersections of each of the direct rows and columns of the ones you marked.

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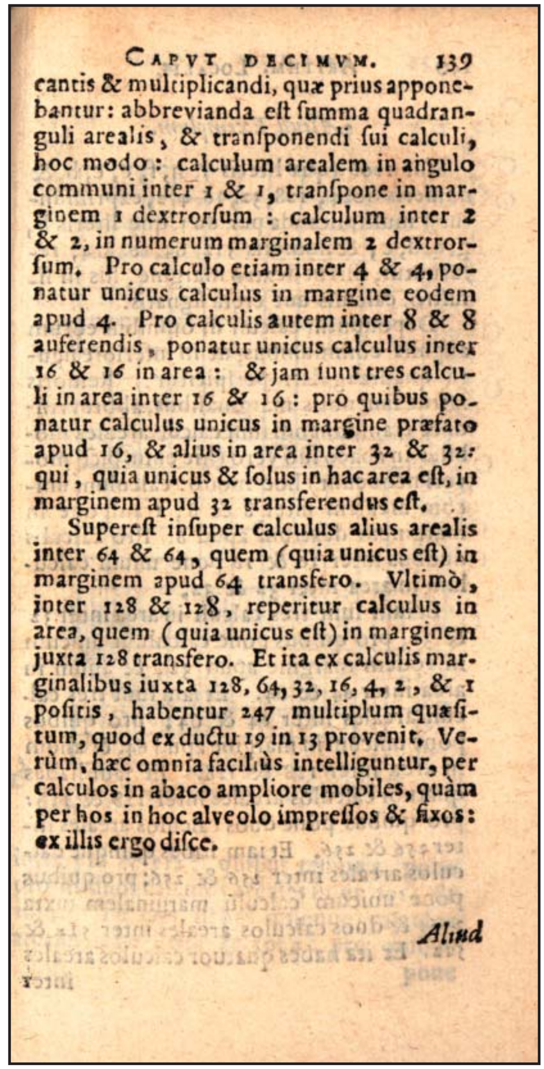
calculi areales figuram quadrangularem exactè referentes quæsitum multiplum, seu productum optatum designant: quod abbreviatione, translatione, & reductione manifestè patebit.

Vt sint multiplicanda 19 (quæ translata fiunt a b e) in 13: quæ translata fiunt a c d. Calculis aut cretâ signentur a b e, vel sui numeri 1, 2, 16, in infimo & sinistro margine ♈ ♉: a c d verò, vel sui numeri 1, 4, 8, in dextro ♈ ♋ signentur, ut infrà. Deinde omnes communes anguli inter sinistras notas a b e, vel 1, 2, 16, & dextras a c d, vel 1, 4. 8, signentur calculis in area depositis, & figuram quadrangularem appositam referent. Abstractis igitur calculis marginalibus, & deletis notis multiplican-



CAPVT DECIMVM. 139

cantis & multiplicandi, quæ prius apponebantur: abbrevianda est summa quadranguli arealis, & transponendi sui calculi, hoc modo: calculum arealem in angulo communi inter 1 & 1, transpone in marginem 1 dextrorsum: calculum inter 2 & 2, in numerum marginalem 2 dextrorsum. Pro calculo etiam inter 4 & 4, ponatur unicus calculus in margine eodem apud 4. Pro calculis autem inter 8 & 8 auferendis, ponatur unicus calculus inter 16 & 16 in area: & jam sunt tres calculi in area inter 16 & 16: pro quibus ponatur calculus unicus in margine præfato apud 16, & alius in area inter 32 & 32: qui, quia unicus & solus in hac area est, in marginem apud 32 transferendus est.

Superest insuper calculus alius arealis inter 64 & 64, quem (quia unicus est) in marginem apud 64 transfero. Vltimò, inter 128 & 128, reperitur calculus in area, quem (quia unicus est) in marginem juxta 128 transfero. Et ita ex calculis marginalibus iuxta 128, 64, 32, 16, 4, 2, & 1 positis, habentur 247 multiplum quæsitum, quod ex ductu 19 in 13 provenit. Verùm, hæc omnia faciliùs intelliguntur, per calculos in abaco ampliore mobiles, quàm per hos in hoc alveolo impressos & fixos: ex illis ergo disce.

Aliud

Napier provides the example of 19 times 13. The first number (19) is marked out on the margin of the abacus (in this case he notes this by the letters a,b,e on the ♈ to ♉ margin) and the second (13) by the letters a,c,d on the ♈ to ♋ margin. Counters are then placed on the intersecting squares. One must now abbreviate the total shown on the abacus (get it into its canonical form). This is accomplished by moving counters as follows:

The single counter found on the intersection of *a* and *a* is moved to the margin (see the counters at the extreme right hand side of the above diagram).

Similarly the single counter found between the margins labeled "2" is moved over, and the single counter between the "4" marks is moved as well.

There are two counters found between the points marked "8" and these are removed and one counter placed on the board in an empty 16 position—there are now three counters in the "16" position.

Two of the counters found at the "16" position are removed and one placed on and empty square in a "32" position while the remaining one is moved to the margin in the "16" position.

Move the single counters in the "32," "64," and "128" positions to the margin.

The margin will now contain counters in the 1, 2, 4, 16, 32, 64, and 128 locations which yields the product of 19 times 13 = 247.

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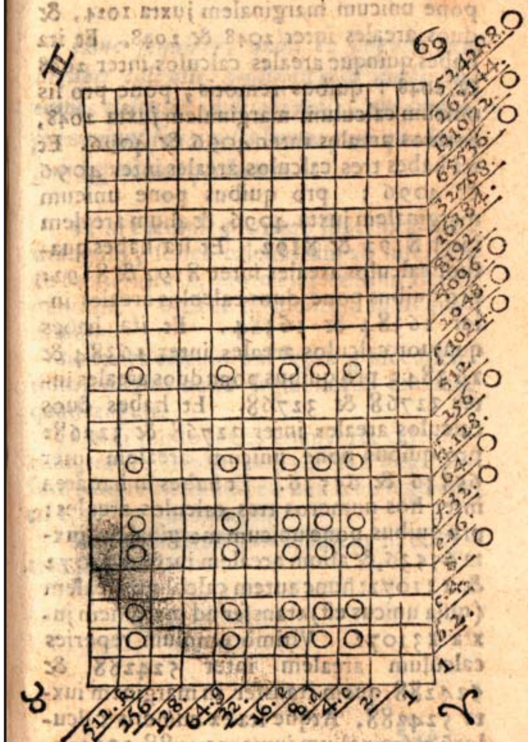
Aliud Exemplum.

Sint 1206 (quæ literis l, h, f, e, c, b, & numeris 1024, 128, 32, 16, 4, 2, exprimuntur) multiplicanda per 604, quę literis k, g, e, d, c, & numeris 512, 64, 16, 8, 4, referuntur, illis in dextro margine, his in sinistro, calculis aut cretâ signatis.

Deponuntur calculi in omnibus eorum angulis communibus; ut in inferiore figura quadrangulari perspicitur. Remotis igitur iam notis marginalibus, abbreviandi & transponendi sunt calculi areales mobiles in abaco suo vero (hic enim picti moveri nequeunt) hoc modo: calculum unicum arealem inter 8 & 8, transpone in marginem dextrum apud 8. Pro calculis duobus inter 16 & 16 pone unum calculum in area inter 32 & 32.

Et iam sunt tres calculi in area inter 32 & 32; pro quibus pone calculum unicum in eodem margine apud 32, & alium in area inter 64 & 64. Et iam sunt tres calculi areales inter 64 & 64: pro quibus pone unicum in margine apud 64, & alium in area inter 128 & 128. Et iam habes quatuor calculos areales inter 128 & 128: pro quibus pone duos calculos areales inter 256 & 256. Et iam habes quinque calculos areales inter 256 & 256: pro quibus pone unicum calculū marginalem iuxta 256: & duos calculos areales inter 512 & 512. Et ita habes quatuor calculos areales inter



inter 512 & 512: quibus remotis, pone pro iis duos calculos areales inter 1024 & 1024. Et iam habes quinque calculos areales inter 1024 & 1024: pro quibus pone

Here Napier provides a second example of multiplying 1,206 times 604 (= 728,424).

pone unicum marginalem juxta 1024, & duos areales inter 2048 & 2048. Et ita habes quinque areales calculos inter 2048 & 2048: quibus remotis, pone pro iis unicum calculum marginalem juxta 2048, & duos areales inter 4096 & 4096. Et ita habes tres calculos areales inter 4096 & 4096: pro quibus pone unicum marginalem juxta 4096, & alium arealem inter 8192 & 8192. Et ita habes quatuor calculos areales inter 8192 & 8192: pro quibus pone duos calculos areales inter 16384 & 16384. Et ita habes quatuor calculos areales inter 16384 & 16384: pro quibus pone duos areales inter 32768 & 32768. Et habes duos calculos areales inter 32768 & 32768: pro quibus pone unicum arealem inter 65536 & 65536. Et habes in hac area inter hos numeros tres calculos areales: pro quibus pone unicum marginalem juxta 65536, & alium arealem inter 131072 & 131072: hunc autem calculum arealem (quia unicus est) transfer ad marginem juxta 131072. Vltimò omnium reperies calculum arealem inter 524288 & 524288, quem transfer in marginem iuxta 524288. Atque ita ex numeris calculorũ marginalium iuxta 524288, 131072, 65536, 4096, 2048, 1024, 256, 64, 32, & 8 collectis in unum, habes 728424 pro multiplo quęsito, quod ex ductu 1206 in 604 provenit.

Hinc

Hinc sequitur, quòd ex singulis quibuslibet calculis multiplicandi ductis in omnes calculos multiplicantis, aut contrà, proveniunt series calculorum quas quadranguli segmenta appellamus.

Vt in exempli proximè superioris quadrangulo, series calculorum ab inferiore & sinisteriore k, motu elephantino ascendentium, dicitur segmentum illius quadranguli.

Sic series calculorum supra g ascendentium, dicitur aliud segmentum eiusdem quadranguli.

Simili modo series transversa calculorum, motu elephantino versus l dextrorsum progredientium, est unum ex segmentis ejusdem quadranguli.

Sic etiam series quæ tendit in h, & cętere similes.

CAP.

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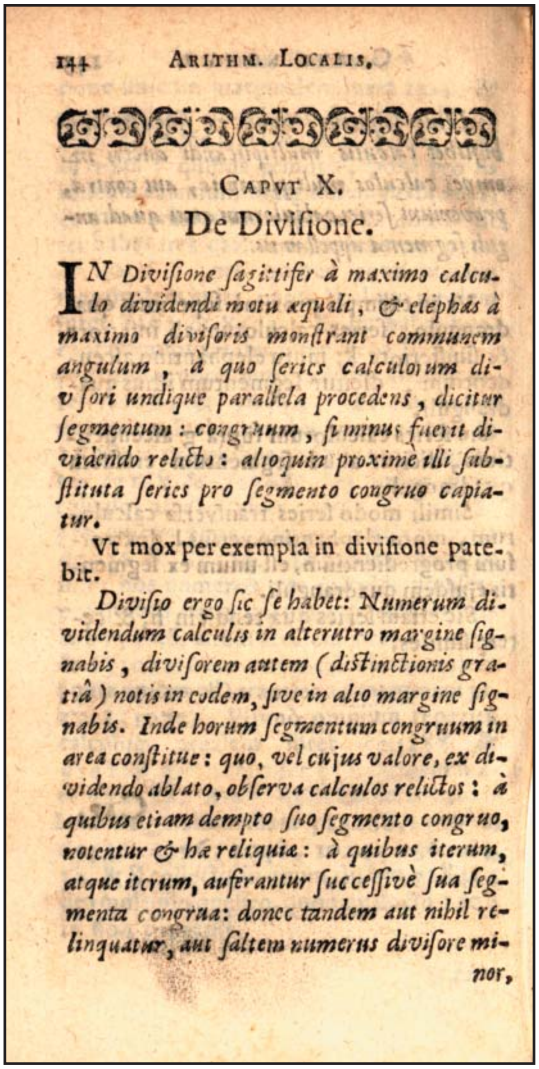

144 ARITHM. LOCALIS.

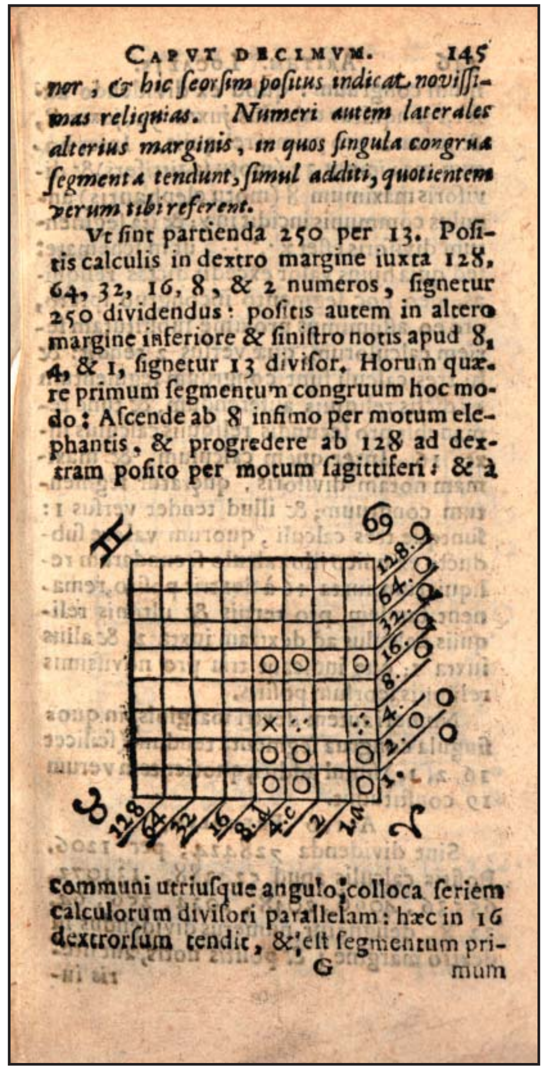
CAPVT X.

De Divisione.

IN Divisione sagittifer à maximo calculo dividendi motu æquali, & elephas à maximo divisoris monstrant communem angulum, à quo series calculorum divisori undique parallela procedens, dicitur segmentum: congruum, si minus fuerit dividendo relicto: alioquin proxime illi substituta series pro segmento congruo capiatur.

Vt mox per exempla in divisione patebit.

Divisio ergo sic se habet: Numerum dividendum calculis in alterutro margine signabis, divisorem autem (distinctionis gratiâ) notis in eodem, sive in alio margine signabis. Inde horum segmentum congruum in area constitue: quo, vel cujus valore, ex dividendo ablato, observa calculos relictos: à quibus etiam dempto suo segmento congruo, notentur & hæ reliquiæ: à quibus iterum, atque iterum, auferantur successivè sua segmenta congrua: donec tandem aut nihil relinquatur, aut saltem numerus divisore minor,


CAPVT DECIMVM. 145

nor; & hic seorsim positus indicat novissimas reliquias. Numeri autem laterales alterius marginis, in quos singula congrua segmenta tendunt, simul additi, quotientem verum tibi referent.

Vt sint partienda 250 per 13. Positis calculis in dextro margine iuxta 128, 64, 32, 16, 8, & 2 numeros, signetur 250 dividendus: positis autem in altero margine inferiore & sinistro notis apud 8, 4, & 1, signetur 13 divisor. Horum quære primum segmentum congruum hoc modo: Ascende ab 8 infimo per motum elephantis, & progredere ab 128 ad dextram posito per motum sagittiferi: & à

communi utriusque angulo, colloca seriem calculorum divisori parallelam: hæc in 16 dextrorsum tendit, & est segmentum primum

G

Chapter X: Division.

Napier's division example (250/13) is shown in the diagram on page 145. The dividend (250) is set up on the ♈ to ♋ margin and the divisor (13) on the ♈to ♉ margin. Beginning with the largest position of the divisor (8) move to directly adjacent squares (in the diagram as shown it is "up") until arriving at the diagonal line denoted by the largest position of the dividend (in this case 128). Place counters on every position from the square thus found (square 8,16 in this case) that correspond to places in the divisor (thus counters go on the 8,16; 4,16; and 1,16 squares.

Subtract the value thus obtained (16*8 + 16*4 + 16 * 1 = 208) from the dividend leaving 42 (which results in marginal counters in positions 32, 8, and 2).

Similar to the first operation, take the largest value in the dividend (8) and move "up" until hitting the diagonal row containing the largest position in the remainder found in the last subtraction (32). Put counters on each square in the row from this location (marked with and "x") that correspond to each position in the divisor (that results in counters in the squares marked "x", "three dots" and "four dots"). The value of this number (52) is greater than the remainder found above (42) so move these counters down one row (from the row marked with an "x" to the row immediately below) and try to subtract this new number (26) from the original remainder (42) which now gives a new remainder of 42 - 26 = 16.

Repeat these operations until the quotient is found by noting the values being "pointed to" by the rows of remaining counters in the center of the abacus (16, 2, 1) and the remainder, if any, will be on the right hand margin.

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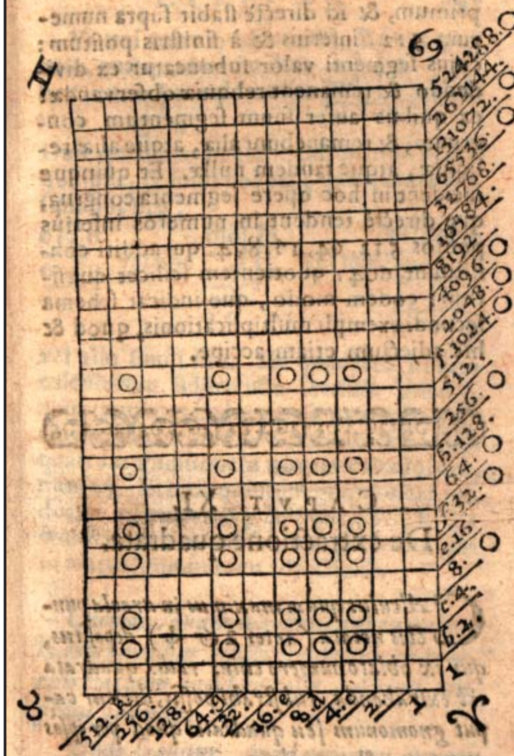
mum congruum: quod ex dividendo aufer, relinquitur calculus juxta 32, juxta 8, & juxta 2 pro primis reliquiis. Inter horum maximum 32 (motu ſagittiferi) & diviſoris maximum 8 (motu elephantis) angulus communis incidit in x; & ita ſegmentum diviſoris eſſet x ·.· ·:· , ut in ſchemate: ſed quia hujus valor excedit dictas reliquias, ideo hoc ſegmento incongruo ſpreto, pro eo aſſumimus proximè ſubſtitutam ſeriem calculorum, quæ verſus 2 tendit: & hi tres calculi ſunt congruum ſegmentum ex dictis reliquiis auferendum, & tunc remanebit pro ſecundis reliquiis calculus iuxta 16. Inter quem calculum, & maximam notam diviſoris, quæratur ſegmentum congruum; & illud tendet verſus 1: ſuntque tres calculi, quorum valore ſubducto ex unico illo calculo ſecundarum reliquiarum, iuxta 16 à dextris poſito, remanent tandem pro tertiis & ultimis reliquiis, calculus ad dextram iuxta 2, & alius iuxta 1; quę indicant tria pro noviſſimis reliquiis ſeorſum poſitis.

Numeri autem dextri marginis, in quos ſingula congrua ſegmenta tendunt (ſcilicet 16, 2, 1,) ſimul additi, quotientem verum 19 conſtituunt.

ALIVD EXEMPLVM.

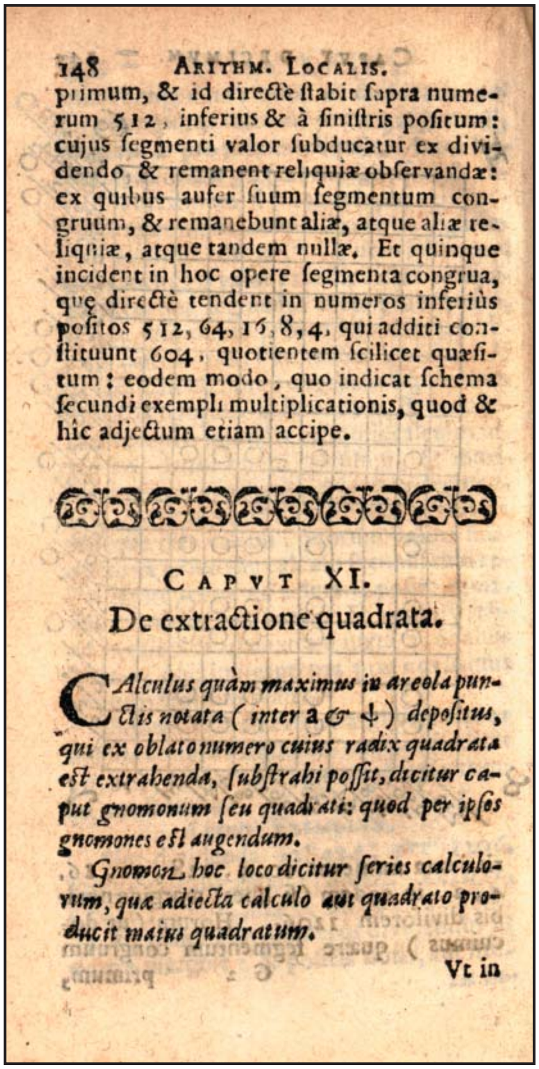
Sint dividenda 728424, per 1206. Poſitis calculis apud 524288, 131072, 65536, 4096, 2048, 1024, 256, 64, 32, 8, deſignetur numerus dividendus in dextro margine: & poſitis notis, aut literis iu-



ris iuxta numeros 1024, 128, 32, 16, 4, 2, in eodem (ſi libet) margine notabis diviſorem 1206. Horum (ut docuimus) quære ſegmentum congruum

 primum,

A second example shows the division 728,424/1206 = 604.



148 ARITHM. LOCALIS.

primum, & id directè ſtabit ſupra numerum 512, inferius & à ſiniſtris poſitum: cujus ſegmenti valor ſubducatur ex dividendo & remanent reliquiæ obſervandæ: ex quibus aufer ſuum ſegmentum congruum, & remanebunt aliæ, atque aliæ reliquiæ, atque tandem nullæ. Et quinque incident in hoc opere ſegmenta congrua, quę directè tendent in numeros inferiùs poſitos 512, 64, 16, 8, 4. qui additi conſtituunt 604, quotientem ſcilicet quæſitum: eodem modo, quo indicat ſchema ſecundi exempli multiplicationis, quod & hic adjectum etiam accipe.

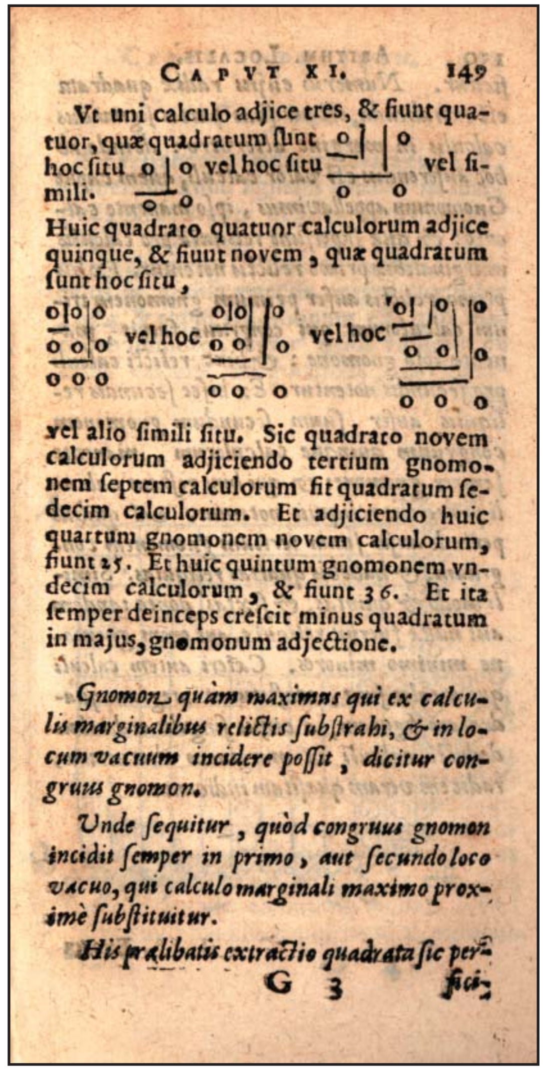
CAPUT XI.

De extractione quadrata.

Calculus quàm maximus in areola punctis notata (inter a & ↓) depoſitus, qui ex oblato numero cuius radix quadrata eſt extrahenda, ſubſtrahi poſſit, dicitur caput gnomonum ſeu quadrati: quod per ipſos gnomones eſt augendum.

Gnomon hoc loco dicitur ſeries calculorum, quæ adiecta calculo aut quadrato producit maius quadratum.

Vt in



CAPUT XI. 149

Vt uni calculo adjice tres, & fiunt quatuor, quæ quadratum ſunt hoc ſitu vel hoc ſitu vel ſimili.

Huic quadrato quatuor calculorum adjice quinque, & fiunt novem, quæ quadratum ſunt hoc ſitu, vel hoc vel hoc vel alio ſimili ſitu. Sic quadrato novem calculorum adjiciendo tertium gnomonem ſeptem calculorum fit quadratum ſedecim calculorum. Et adjiciendo huic quartum gnomonem novem calculorum, fiunt 25. Et huic quintum gnomonem vndecim calculorum, & fiunt 36. Et ita ſemper deinceps creſcit minus quadratum in majus, gnomonum adjectione.

Gnomon quàm maximus qui ex calculis marginalibus relictis ſubſtrahi, & in locum vacuum incidere poſſit, dicitur congruus gnomon.

Unde ſequitur, quòd congruus gnomon incidit ſemper in primo, aut ſecundo loco vacuo, qui calculo marginali maximo proximè ſubſtituitur.

His prælibatis extractio quadrata ſic perfici-

G 3

Chapter XI: Calculation of the square root.

This process requires one to add counters to the abacus (board) to make square figures. The top of page 149 shows diagrams that explain this process. Begin by placing a single counter on the board (it will actually go on one of the dotted squares). Adding three other counters adjacent (or with blank rows and columns between them and the first one placed) will result in another square figure on the abacus. Similarly adding another five counters to this (with or without the blank rows and columns shown) will result in an even bigger square.

Take the number to be considered and put counters along one margin that represent its value.

From the position of the largest counter in that value, follow the diagonal lines (bishop's moves) across the board until you come to a square with a dot. Place a counter on that square.

Subtract the value represented by this single counter from the original number in the margin.

Add three (five, seven, ... for subsequent steps) to create a square on the board and subtract the value of the added counters from the number in the margin until the number is either too large to be subtracted or there is no space left on the board. You should be left with a large square of counters (perhaps with blank rows and columns between them) on the board.

Move one of the counters in each row of the square to the margin and the positions of these marginal counters will yield the square root of the number.

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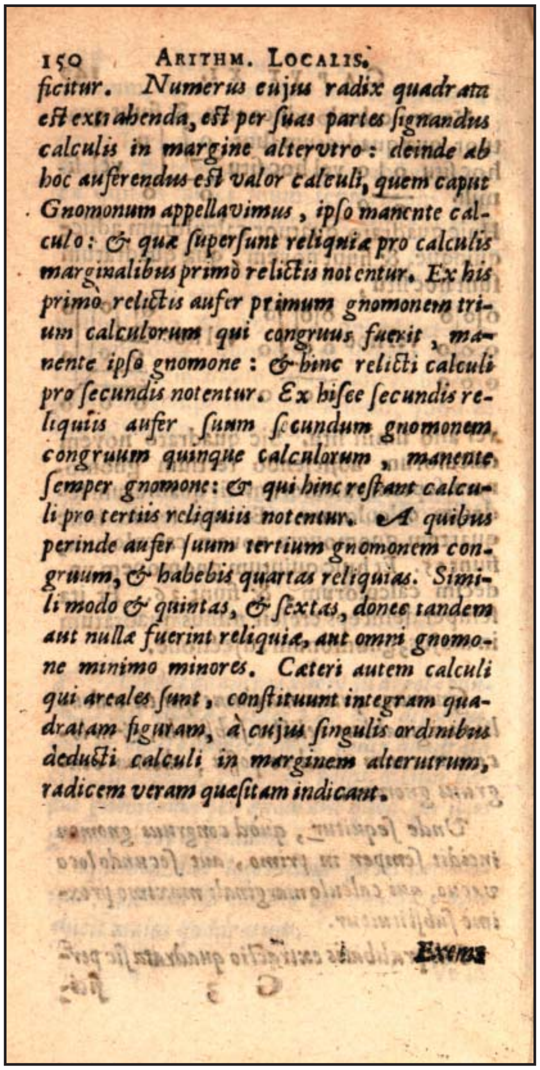
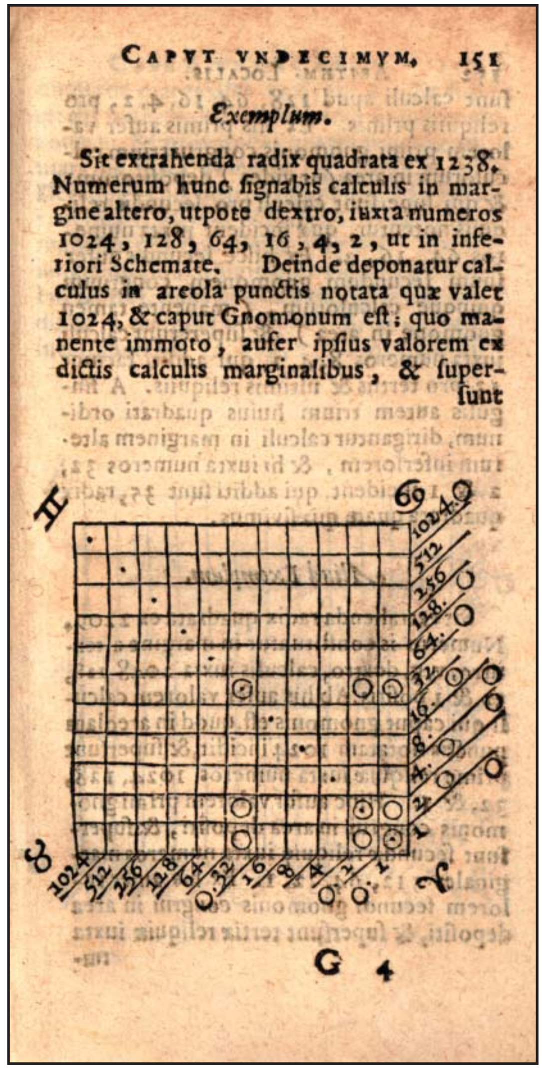


150 ARITHM. LOCALIS.

ficitur. Numerus cujus radix quadrata est extrahenda, est per suas partes signandus calculis in margine altervtro: deinde ab hoc auferendus est valor calculi, quem caput Gnomonum appellavimus, ipso manente calculo: & quæ supersunt reliquiæ pro calculis marginalibus primò relictis notentur. Ex his primo relictis aufer primum gnomonem trium calculorum qui congruus fuerit, manente ipso gnomone: & hinc relicti calculi pro secundis notentur. Ex hisce secundis reliquiis aufer suum secundum gnomonem congruum quinque calculorum, manente semper gnomone: & qui hinc restant calculi pro tertiis reliquiis notentur. A quibus perinde aufer suum tertium gnomonem congruum, & habebis quartas reliquias. Simili modo & quintas, & sextas, donec tandem aut nullæ fuerint reliquiæ, aut omni gnomone minimo minores. Cæteri autem calculi qui areales sunt, constituunt integram quadratam figuram, à cujus singulis ordinibus deducti calculi in marginem alterutrum, radicem veram quæsitam indicant.

Exem-



CAPVT VNDECIMVM. 151

Exemplum.

Sit extrahenda radix quadrata ex 1238. Numerum hunc signabis calculis in margine altero, utpote dextro, iuxta numeros 1024, 128, 64, 16, 4, 2, ut in inferiori Schemate. Deinde deponatur calculus in areola punctis notata quæ valet 1024, & caput Gnomonum est; quo manente immoto, aufer ipsius valorem ex dictis calculis marginalibus, & supersunt

G 4

Napier provides an example of determining the square root of 1238.

The largest counter is in the 1024 position so the first counter is placed on the dot found by moving down the 1024 diagonal (at the 32,32 position). Subtracting this value (1024) from the original number leaves counters at 128, 64, 16, 4 and 2 (= 214).

Placing three counters on the board to form a square with the first counter but whose value can still be subtracted from 214, results in counters at positions 32,2; 2,2; and 2,32 (whose values are 64, 4 and 64, which when subtracted from the remainder of 214 = 82.

The next square that can be constructed from five counters, yet the values of those five counters still being capable of being subtracted from 82 results in counters in positions 32,1; 2,1; 1,1; 1.2; and 1,32. The values of these five counters total 69 which when subtracted from 82 leave 13 as a remainder.

As there is no more room on the board we have to stop.

Move one counter from each row to the margin (rows 32, 2 and 1) and this value (35) is the square root required, or at least the integer part of it (the actual value is 35.1852....).

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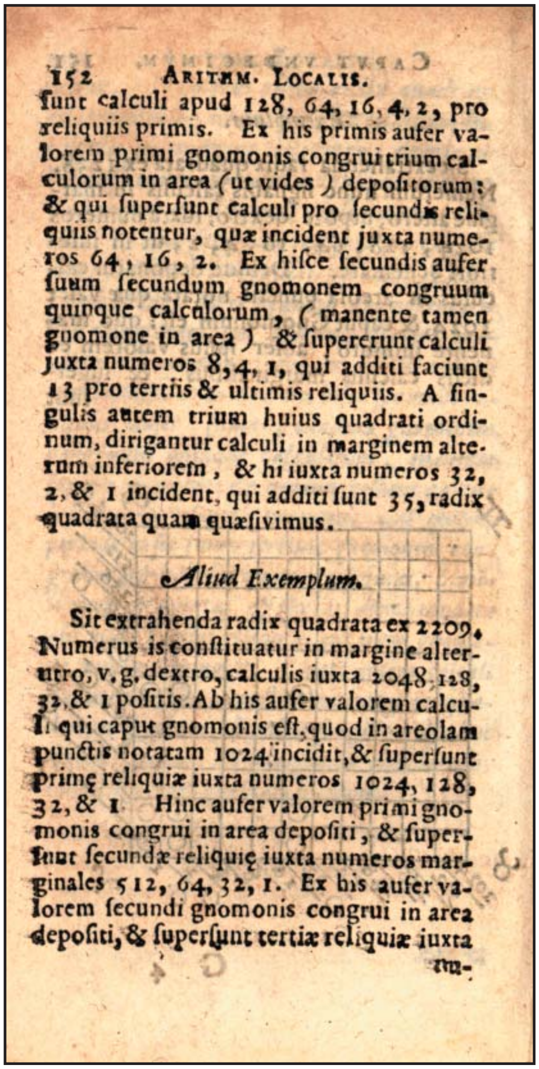
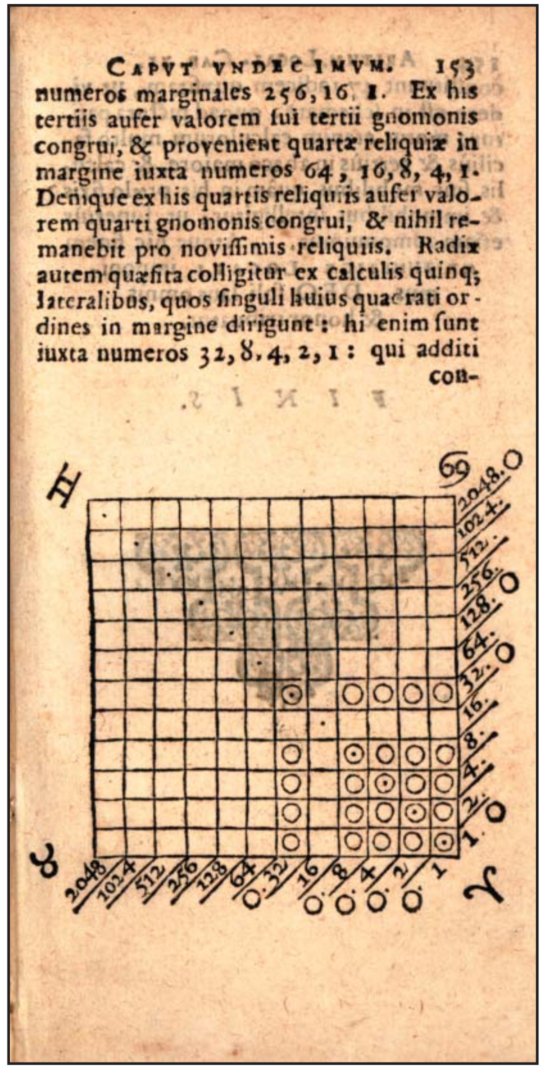


funt calculi apud 128, 64, 16, 4, 2, pro reliquiis primis. Ex his primis aufer valorem primi gnomonis congrui trium calculorum in area (ut vides) depositorum: & qui supersunt calculi pro secundis reliquiis notentur, quæ incident juxta numeros 64, 16, 2. Ex hisce secundis aufer suum secundum gnomonem congruum quinque calculorum, (manente tamen gnomone in area) & supererunt calculi juxta numeros 8, 4, 1, qui additi faciunt 13 pro tertiis & ultimis reliquiis. A singulis autem trium huius quadrati ordinum, dirigantur calculi in marginem alterum inferiorem, & hi iuxta numeros 32, 2, & 1 incident, qui additi sunt 35, radix quadrata quam quæsivimus.

Aliud Exemplum.

Sit extrahenda radix quadrata ex 2209. Numerus is constituatur in margine alterutro, v. g. dextro, calculis iuxta 2048, 128, 32, & 1 positis. Ab his aufer valorem calculi qui caput gnomonis est, quod in areolam punctis notatam 1024 incidit, & supersunt primę reliquiæ iuxta numeros 1024, 128, 32, & 1. Hinc aufer valorem primi gnomonis congrui in area depositi, & supersunt secundæ reliquię iuxta numeros marginales 512, 64, 32, 1. Ex his aufer valorem secundi gnomonis congrui in area depositi, & supersunt tertiæ reliquiæ iuxta nu-



numeros marginales 256, 16, 1. Ex his tertiis aufer valorem sui tertii gnomonis congrui, & provenient quartæ reliquiæ in margine iuxta numeros 64, 16, 8, 4, 1. Denique ex his quartis reliquiis aufer valorem quarti gnomonis congrui, & nihil remanebit pro novissimis reliquiis. Radix autem quæsita colligitur ex calculis quinq; lateralibus, quos singuli huius quadrati ordines in margine dirigunt: hi enim sunt iuxta numeros 32, 8, 4, 2, 1: qui additi con-

Napier provides a second example for calculating the square root of 2209 (= 47).

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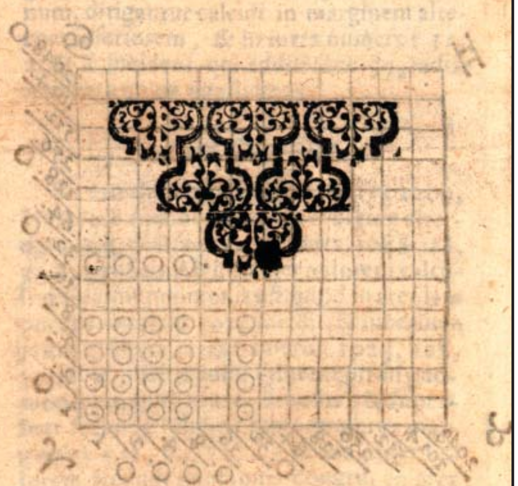
constituunt 47 radicem quæsitam, ut videre est in schemate, quoad pictura patitur: motus etenim calculorum multò facilius & certius in abaco maiore, & calculis suis mobilibus, quàm in his prælo fixis & immobilibus intelligitur; ut superius etiam admonuimus. Atque hic finem ARITHMETICÆ LOCALI imponimus. DEO soli laus omnis & honor tribuatur.

FINIS.



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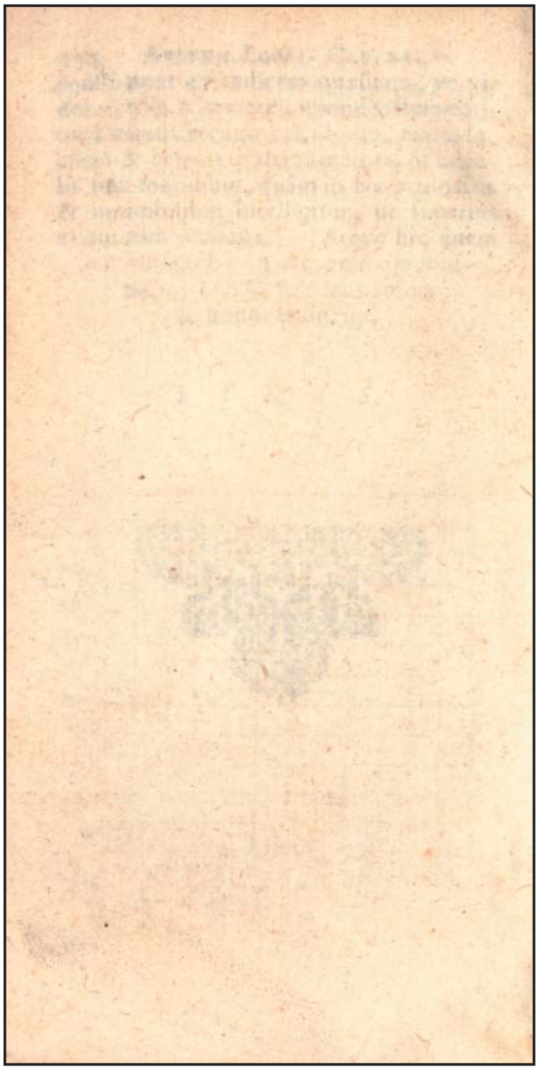

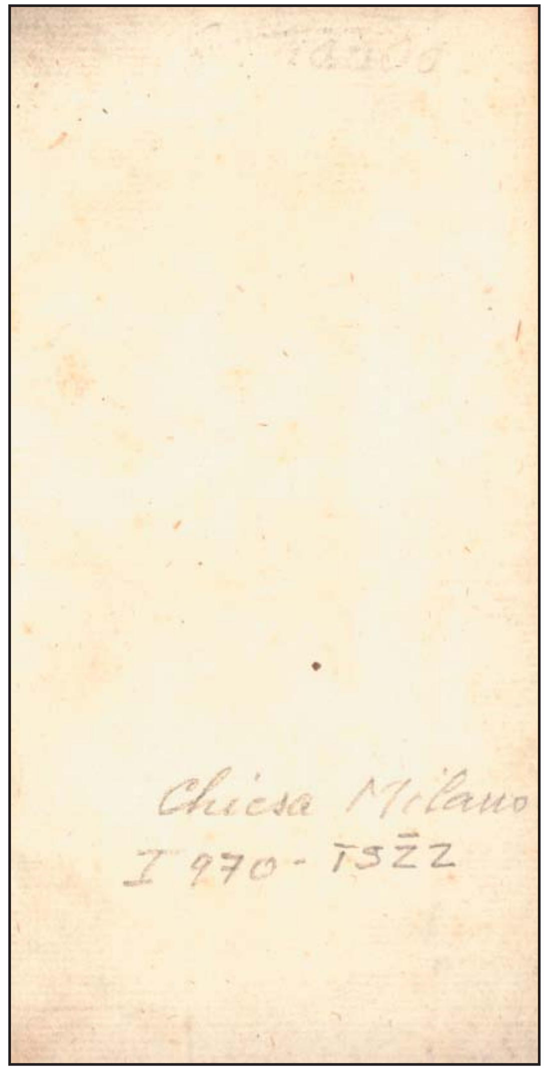

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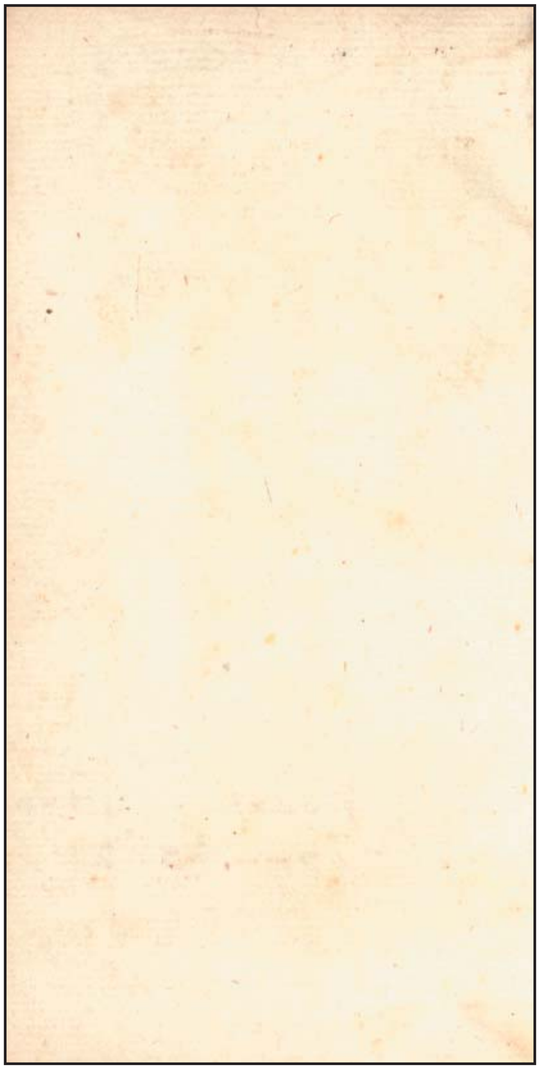

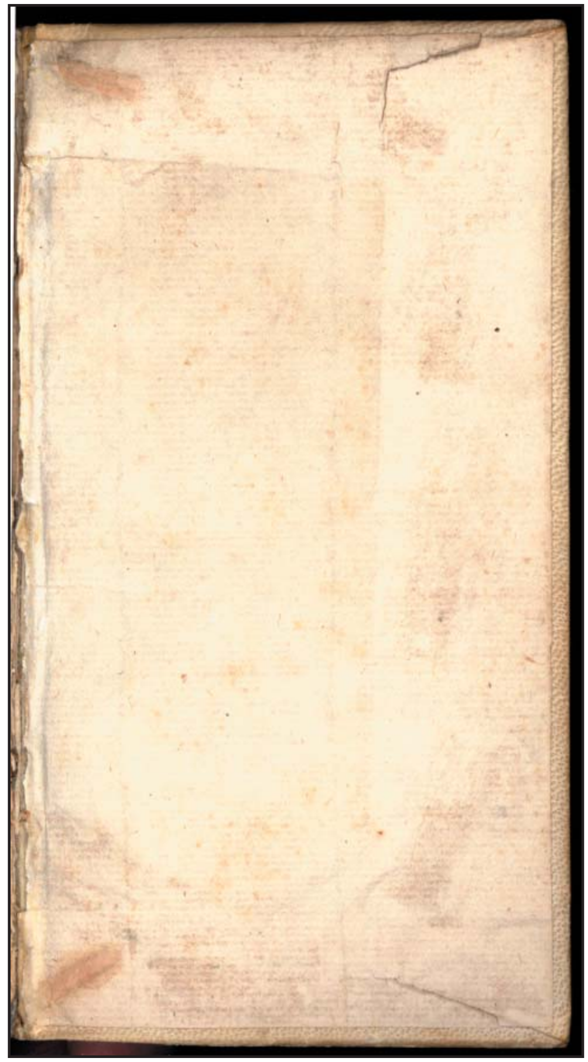

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